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Local spectra in plane channel flow using wavelets designed for the interval

M. Uhlmann[1] and J. Fröhlich[2]

[1]Potsdam Institute For Climate Impact Research,
14412 Potsdam, Germany.
[2]Institute for Hydromechanics, University of Karlsruhe,
76128 Karlsruhe, Germany.

Contact address: *uhlmann@pik-potsdam.de*

1 Introduction

The wavelet transform allows for a decomposition of signals with respect to position *and* scale. It is an attractive formalism for the analysis of turbulent flows because turbulent signals in space and time contain statistically significant localized features, so-called coherent structures.

In the past, the wavelet analysis of flows in bounded domains has been hampered by the lack of suitable analyzing functions for closed intervals. Typically, studies of the flow scales in turbulent plane channels have been restricted to the analysis of the doubly-periodic wall-parallel planes (e.g. [1]). This does not allow to explicitly account for phenomena in the wall-normal direction. When using proper orthogonal decomposition in the inhomogeneous direction, no clear *a priori* definition of "physical length scale" is available for the modes. Since there is a continuing scientific and technological interest in understanding the dynamics of near-wall turbulence we wish to extend existing wavelet instruments such that they can deal with the bounded coordinate direction.

The problem with existing wavelet methods for treating the interval is that they do not meet all of the criteria required for the purpose of analysis of data from DNS of turbulent flow: (i) Discrete orthogonal transform. It allows for filtering in wavelet space and does not increase the data size. (ii) Orthogonality with respect to a scalar product with unity weight. The total energy then is represented by the sum of the squares of the wavelet coefficients (Parseval's theorem). (iii) Symmetry of wavelet functions, at least in the center of the interval. (iv) An acceptable trade-off between localization in space and scale. More specifically, Daubechies wavelets for the interval [2] violate condition (iii); mapping the interval to a periodic space, as in [3], does not allow for (ii) [4].

2 New orthogonal polynomial basis

In the present study we employ a new orthogonal wavelet basis, developed in [4], which is generated from a recombination of Legendre polynomials L_k. Chebyshev polynomials of the second kind, U_k, evaluated at a suitable set of root points $y_l^{(m)}$ are used as weights [5]:

$$\psi_{ji}(x)=C_{ij}^{\psi}\sum_{k=2^j+1}^{2^{j+1}}U_k(y_i^{(2^j)})\sqrt{k+1/2}\cdot L_k(x)\,,\quad j=0,1,\ldots\quad i=0\ldots 2^j\!-\!1\,. \tag{1}$$

where C_{ij}^{ψ} are normalization constants. The characteristic length scale of each function ψ_{ji} varies with position, i.e. there is no strict translational invariance. Hence, the physical scale of each wavelet is measured by an appropriately defined scale function $s_x = s(j,i)$.

We have constructed non-classical multi-resolution analyses for the treatment of two-dimensional data with one periodic (x) and one bounded coordinate direction (y), i.e. for spanwise or streamwise slices extracted from plane channel flow. Higher dimensions or other combinations are straightforward. Here, only one variant is considered where a tensor product between periodic spline wavelets $\tilde{\psi}_{j_x,i_x}(x)$ and the new polynomial wavelets $\psi_{j_y,i_y}(y)$ is performed with independent scale indices j_x, j_y for the two directions. The decomposition of a scalar signal then reads:

$$u(x,y) = \text{scaling functions} + \sum_{j_x=0}^{J}\sum_{j_y=0}^{J}\sum_{i_x=0}^{2^{j_x}-1}\sum_{i_y=0}^{2^{j_y}-1} u_{i_x,i_y}^{j_x,j_y}\,\tilde{\psi}_{j_x,i_x}(x)\,\psi_{j_y,i_y}(y)\,, \tag{2}$$

where a wavelet coefficient $u_{i_x,i_y}^{j_x,j_y}$ is obtained by the scalar product of the signal and the corresponding wavelet function due to (i).

3 Local wavelet spectra

We consider data from two DNS of plane channel flow performed by the first author at Reynolds numbers $Re_\tau = 190$ and $Re_\tau = 590$ in a domain of size $2\pi \times 2 \times \pi$ using $600 \times 385 \times 600$ discrete Fourier/Chebyshev modes, respectively.

We define the ensemble-averaged 1d power spectral density as a function of wall-normal position y and wall-normal "scale-number" $k_y \equiv 1/s_y$ (the latter being the wavelet analog to a Fourier wavenumber):

$$E_{\alpha\alpha}(y;k_y) \equiv 2^{j_y}\sum_{j_x,i_x}\frac{<\left((u_\alpha)_{i_x,i_y}^{j_x,j_y}\right)^2>}{\Delta k_y}\,, \tag{3}$$

where u_α is a component of the velocity fluctuation. Statistics from 150 planes gathered over one flow-through time have been used to compute the pre-multiplied spectra shown in Figure 1. The large scale limit of all wall-normal spectra is imposed by the channel height of $2h^+ = 2Re_\tau$). At the higher Reynolds number the overall range of active modes is broader, reflecting the widening of the gap between inner and outer scales. For a given Reynolds number, increasing the wall distance means moving the small-scale limit of the pre-multiplied spectrum towards larger scales and steepening it, thereby reducing the range of excited scales. The behavior of the streamwise (u_1) and wall-normal (u_2) components differs considerably. At $Re_\tau = 590$, $k_y E_{22}$ has a peak scale which increases progressively from 15 wall units at $y^+ = 5$ to the full channel height, whereas $k_y E_{11}$ always has the maximum energy content at the largest scale $s_y^+ = 2Re_\tau$. In this respect the pre-multiplied spectra for the spanwise component are somewhat intermediary between u_1 and u_2. Our observations are consistent with experimental two-point correlations of streamwise and wall-normal velocity components in [6]. Moreover, we point out the agreement with Townsend's [7] attached eddy model stating that u_2 is significantly constrained in the logarithmic layer through the impermeability condition of the wall.

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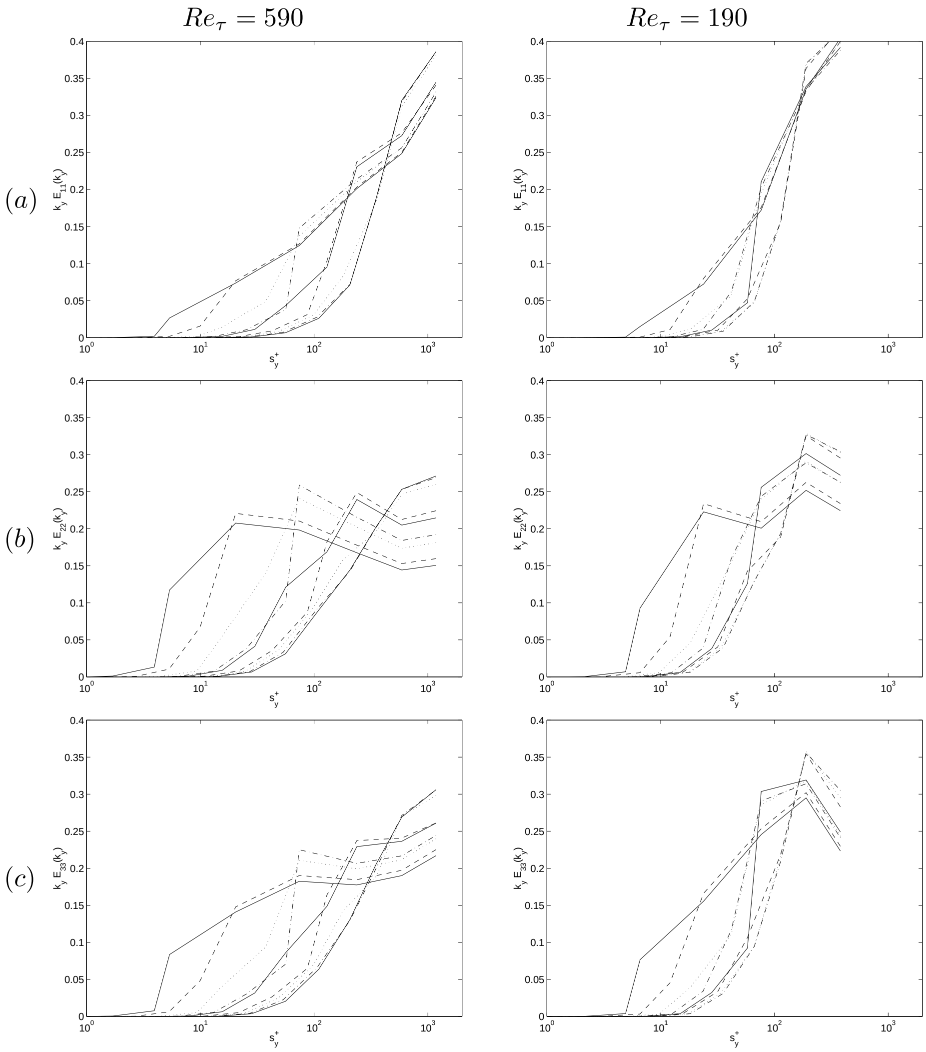

Figure 1: Pre-multiplied wall-normal power spectra $k_y E_{\alpha\alpha}(y^+; k_y)$ as a function of scale s_y^+ in plane channel flow at $Re_\tau = 590$ with $y^+ = \{5, 10, 30, 60, 100, 200, 300, 400, 500\}$(left) and $Re_\tau = 190$ with $y^+ = \{5, 10, 30, 50, 75, 100, 140, 190\}$ (right) for (a) streamwise, (b) wall-normal and (c) spanwise velocity components. Increasing y^+ means a shift towards larger scales while line styles rotate through solid, dashed, dotted, dash-dotted. Spectra are normalized to unit area.