Local Spectra in Plane Channel Flow Using Wavelets Designed for the Interval

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Motivation & Scope

near-wall turbulence dynamics:

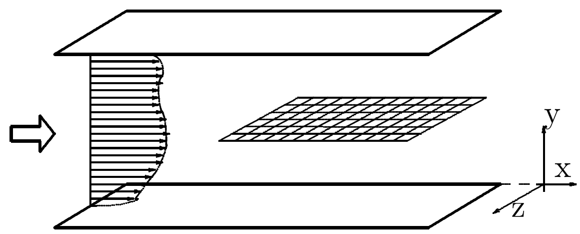
- multi-scale, bearing coherent structures
- bounded coordinate direction

wavelet analysis:

- access to space and scale information

⇒ however, in practice: often limited to wall-parallel planes

wall-normal scales?



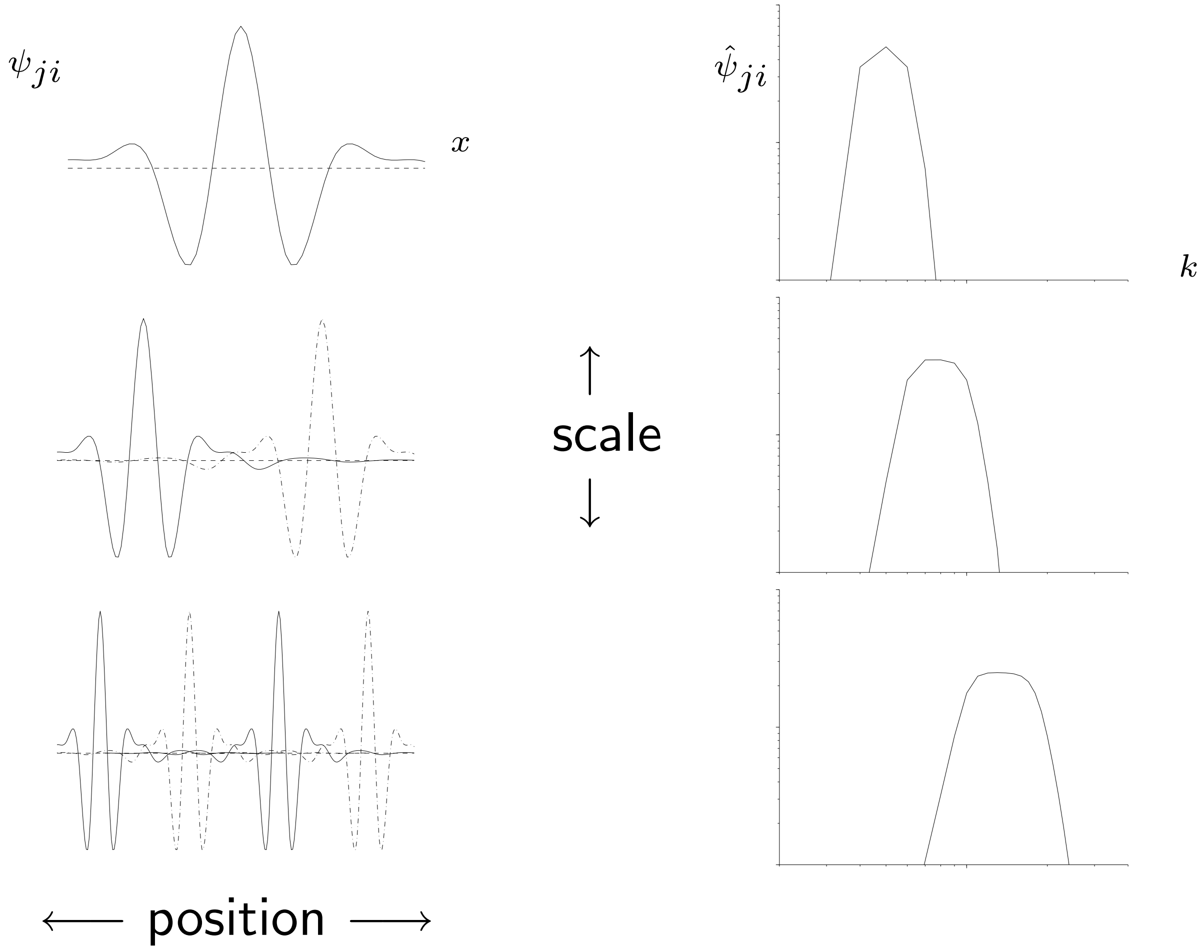
towards:

⋆ adequate basis for the interval

⋆ application to channel flow data

Discrete Orthogonal Wavelets

physical space spectral space



decomposition $$f(x) = \sum_{j,i} \hat{f}_{ji}\,\psi_{ji}(x) \;+\; \text{low-pass}$$

- energy: Parseval
- symmetry
- localization

Legendre Basis for $x \in [-1, 1]$

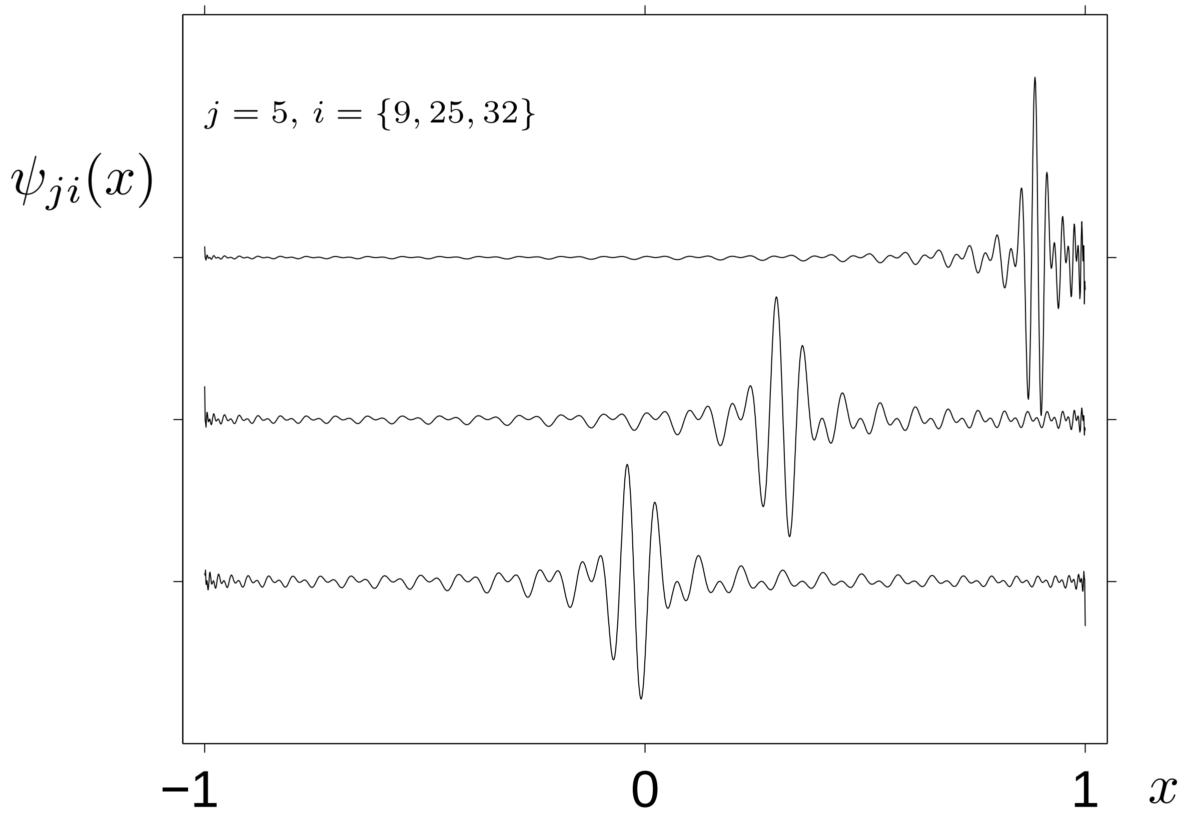
$$\psi_{ji}(x) \sim \sum_{k=2^j+1}^{2^{j+1}} \sin\left(\frac{\pi(k+1)(i+1)}{2^j+1}\right) L_k(x)$$

- sharp cut-off in polynomial space
- scale indices $j \in \mathbb{N}_0$
- shift indices $i = 0 \ldots 2^j - 1$

$\Rightarrow$ orthogonal w.r.t. weight unity

$$\int_{-1}^{+1} \psi_{ji}\,\psi_{kl}\,\mathsf{d}x = \delta_{jk}\,\delta_{il}$$

Legendre Wavelets: Shape

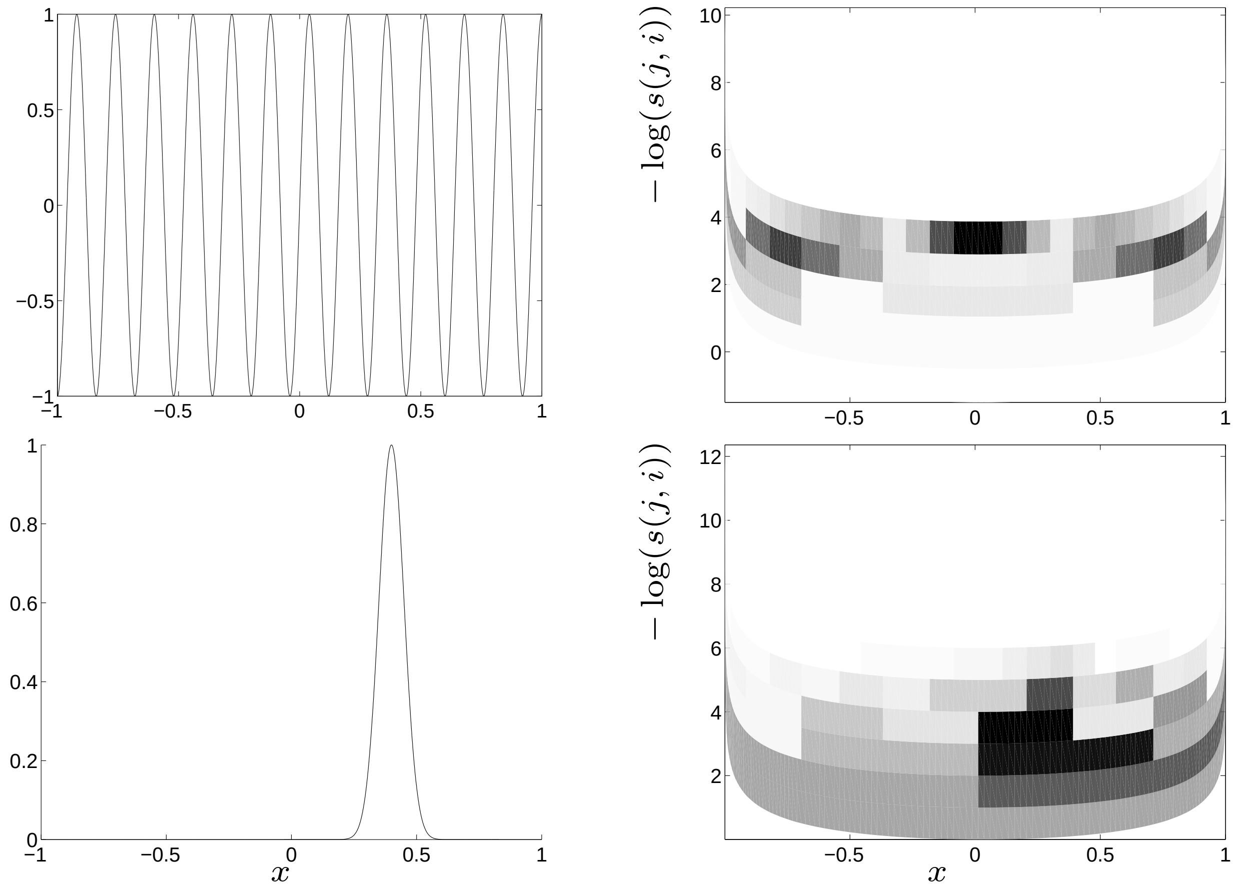


- spatial localization $\sim x^{-1}$
- no translational invariance: characteristic scale depends on position $s(j,i)$
- non-uniform shifts

Artificial Data

coefficient $$\hat{f}_{ji} = \int_{-1}^{+1} f(x)\,\psi_{ji}(x)\,\mathrm{d}x$$

signal $f(x)$ coefficient "scalogram"

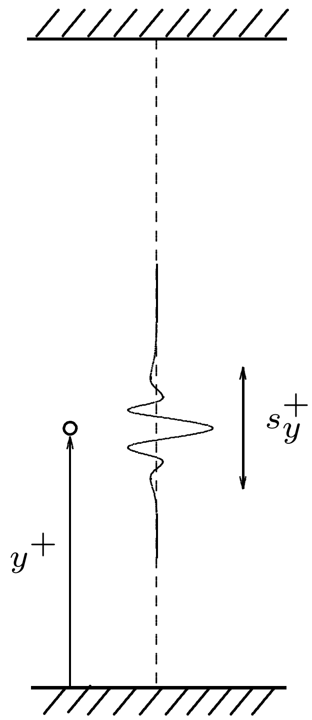


- reading of scale and position

Local energy spectrum

- define power spectral density

$$E_{uu}(k_y; y) \equiv 2^j \frac{\hat{u}_{ji}^2}{\Delta k_y}$$



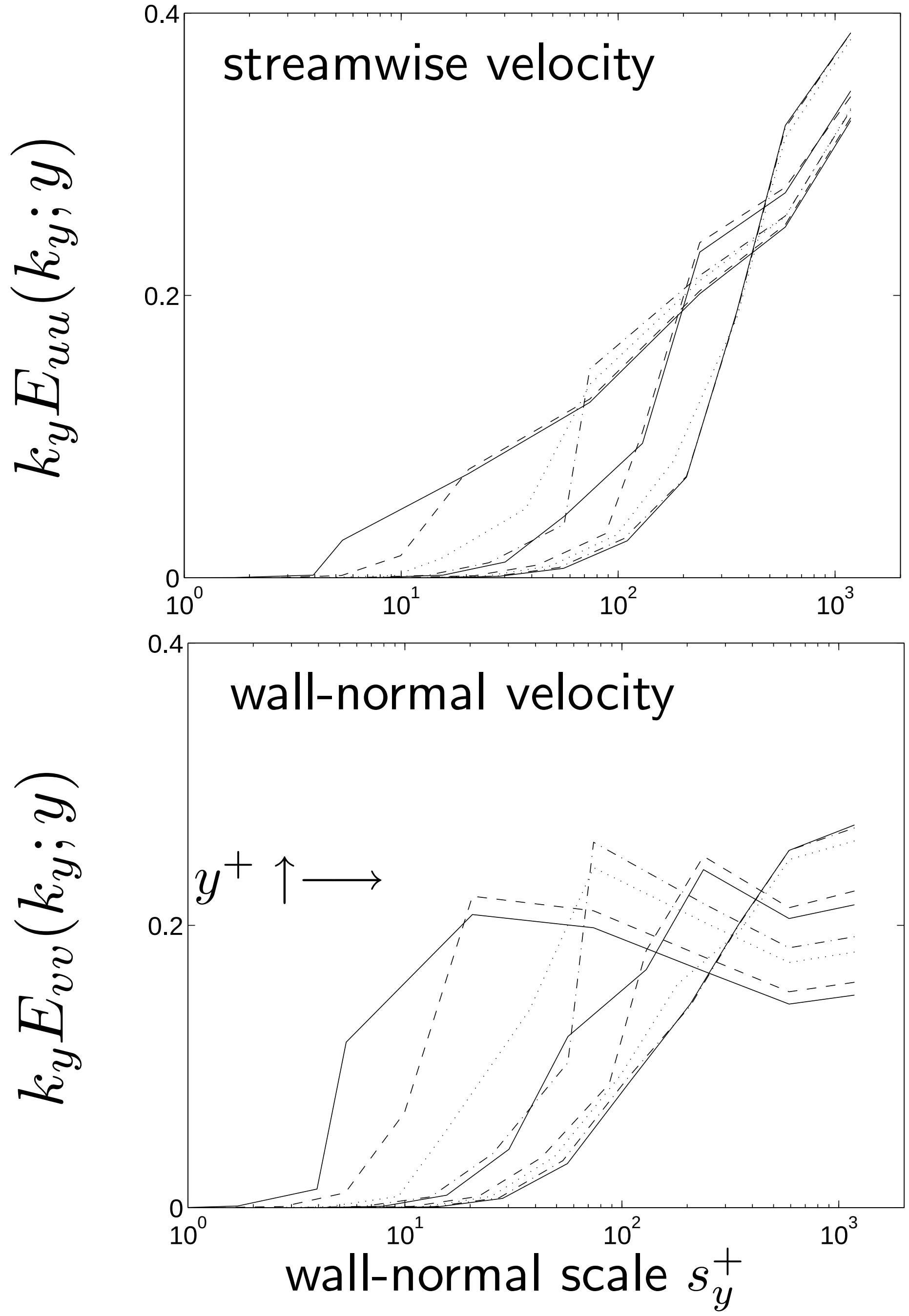
and wave-number or "scale-number"

$$k_y = \frac{1}{s_y}$$

- channel flow DNS: $Re_\tau = 590$, box $2\pi \times 2 \times \pi$, $600 \times 385 \times 600$ modes
- statistics over one "wash-out" cycle

Wall-Normal Spectra

$y^+ = \{5, 10, 30, 60, 100, 200, 300, 400, 500\}$



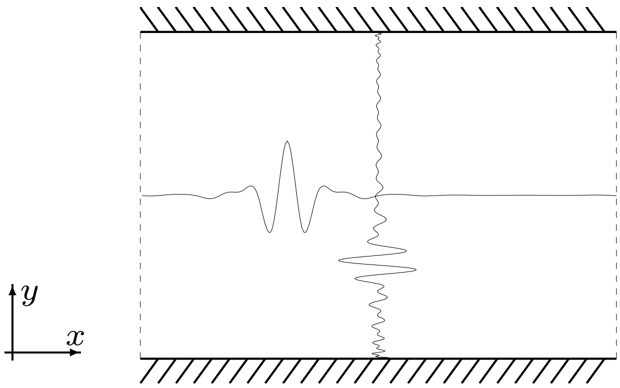
- v-scales constrained

Two-Dimensional Analysis

tensor product of different one-dimensional bases:

$$\psi_{i_x,i_y}^{j_x,j_y}(x,y) = \tilde{\psi}_{j_x,i_x}(x) \cdot \psi_{j_y,i_y}(y)$$

$x \in \mathbb{R}/\mathbb{Z}$	$y \in [-1,+1]$
spline wavelets	Legendre wavelets

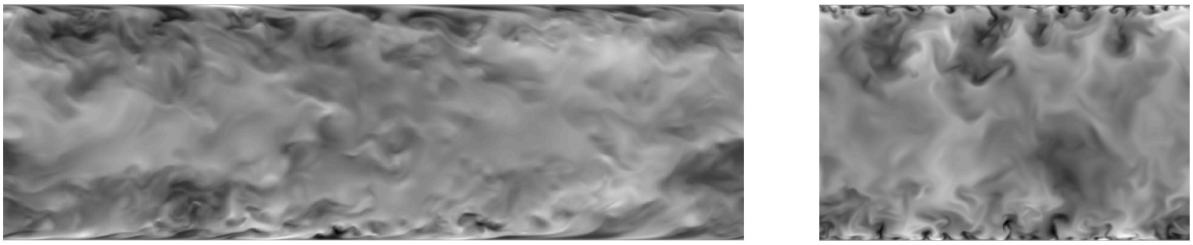


intermittency index:

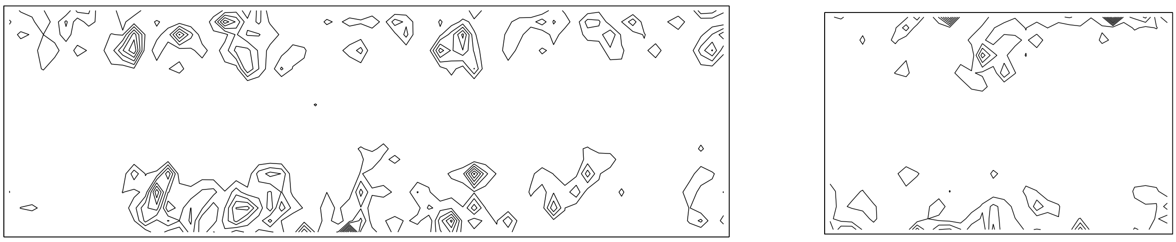
$$I(s_x;s_y;t) = \frac{E_{u_i u_i}(x;y;s_x;s_y;t)}{E_{u_i u_i}(s_x;s_y;t)}$$

Intermittency Index

streamwise component of the signal



$$s_x^+ = s_y^+ \approx 60; \; \max(I) \approx 20$$



$$s_x^+ = s_y^+ \approx 15; \; \max(I) \approx 200$$

streamwise cut spanwise

Conclusion

Legendre-polynomial wavelets

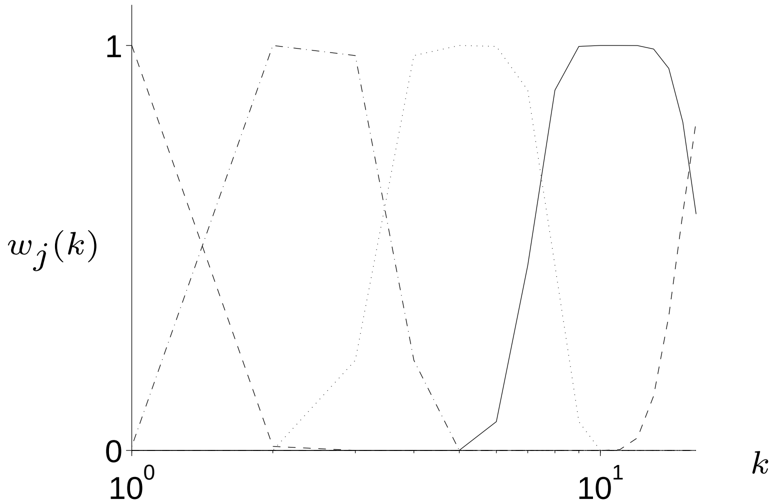
- orthogonal basis w.r.t. weight unity (Parseval)
- useful for data including bounded coordinate direction
- allow for wall-normal local spectra

Perspective

⋆ improve spatial localization

Work in Progress: Localization

smooth spectral windows:



$$\psi_{ji}(x) \sim \sum_{k=0}^{\infty} \underline{w_j(k)}\, b_{j,i}(k)\, L_k(x)$$