Turbulenzmodelle in der Strömungsmechanik

Turbulent flows and their modelling

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RANS modeling
The turbulent viscosity assumption
Conclusion

LECTURE 8

Introduction to RANS modelling

Questions to be answered in the present lecture

How can the Reynolds-averaged equations be closed?

What are the different types of models commonly used?

Do simple eddy viscosity models allow for acceptable predictions?

RANS modeling
The turbulent viscosity assumption
Conclusion

The challenge of turbulence

Recap of the salient features of turbulent flows

- 3D, time-dependent, random flow field
- largest scales are comparable to characteristic flow size
  $\rightarrow$ geometry-dependent, not universal
- wide range of scales: $\tau_\eta / T \sim Re^{-1/2}$, $\eta / L \sim Re^{-3/4}$
- wall flows: energetic motions scale with viscous units
  $\delta_\nu / h \sim Re^{-0.88}$
- non-linear & non-local dynamics

General criteria for assessing turbulence models

Level of description

- how much information can be extracted from the results?

Computational requirements & development time

- how much effort needs to be invested in the solution?

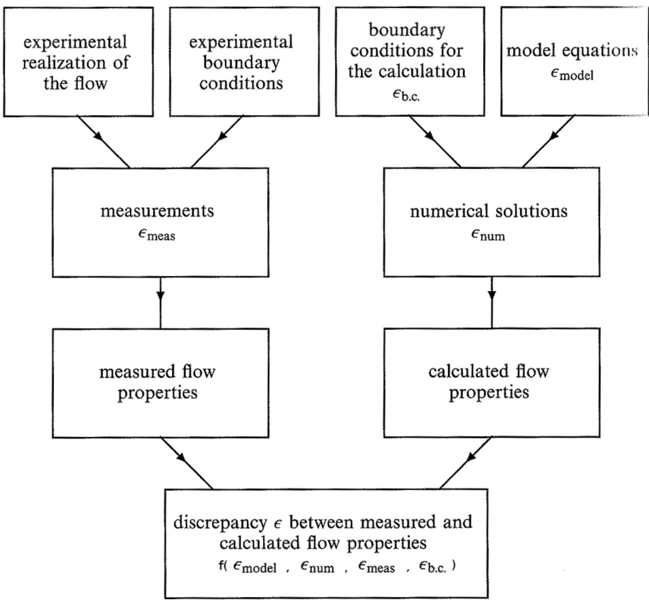
Accuracy

- how precise and trustworthy are the results?

Range of applicability

- how general is the model?

Possible discrepancies between computation & experiment



(adapted from Pope "Turbulent flows")

Reynolds averaging procedure – need for modeling

- decompose velocity field into mean and fluctuation:

$$\mathbf{u}(\mathbf{x},t) = \langle \mathbf{u}(\mathbf{x},t)\rangle + \mathbf{u}'(\mathbf{x},t)$$

- average continuity & momentum equations:

$$\begin{aligned} \langle u_i\rangle_{,i} &= 0 \\ \partial_t\langle u_i\rangle + \left(\langle u_i\rangle\langle u_j\rangle\right)_{,j} + \frac{1}{\rho}\langle p\rangle_{,i} &= \nu\langle u_i\rangle_{,jj} - \underline{\langle u_i' u_j'\rangle_{,j}} \end{aligned}$$

- task of RANS models:

  $\longrightarrow$ supply the unclosed Reynolds stresses $\langle u_i' u_j'\rangle$

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Reynolds averaging – the closure problem

Averaging always introduces more unknowns than equations

- transport equation for the nth moment

$\longrightarrow$ contains $(n+1)$th moment

... and so on

$\Rightarrow$ requires *closure* at some level

- the higher the level, the more terms need modeling

Most successful closures:

- $n = 1$: turbulent viscosity models
- $n = 2$: Reynolds stress models

Common types of RANS models

Models based on the turbulent viscosity hypothesis

$$\langle u_i' u_j' \rangle = -\nu_T \left(\langle u_i \rangle_{,j} + \langle u_j \rangle_{,i} \right) + \tfrac{2}{3} \delta_{ij}\, k$$

- turbulent viscosity ν_T needs to be specified (modeled)

Reynolds-stress transport models

$$\frac{\bar{\mathrm{D}} \langle u_i' u_j' \rangle}{\bar{\mathrm{D}} t} = \ldots$$

- various unknown terms (cf. lecture 10)

Non-linear turbulent viscosity models

$\langle u_i' u_j' \rangle = \text{non-linear-function}\left(\langle u_i \rangle_{,j}, k, \varepsilon, \ldots \right)$ (cf. lecture 12)

Assumptions behind Boussinesq's hypothesis

$$\boxed{\langle u_i' u_j' \rangle - \frac{2}{3} k\, \delta_{ij} = -2 \nu_T \bar{S}_{ij}}$$

Reynolds stress assumed proportional to local mean strain rate

1. mechanisms generating Reynolds stress are assumed local

   $\longrightarrow$ transport effects neglected

2. turbulent stress and mean strain are assumed aligned

   $\longrightarrow$ this stems from the linearity of the relation

$\rightsquigarrow$ assumptions in general <u>not true</u>!

The locality assumption: example of failure

Experiments demonstrate:

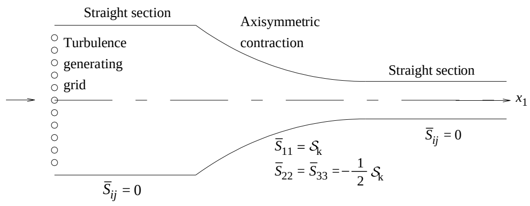
- importance of history effects
- contraction with $\bar{S}_{ij} = cst$

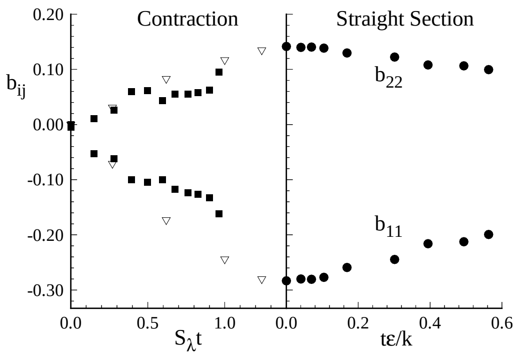
but: increasing anisotropy

- $\bar{S}_{ij} = 0$ in straight section

but: non-zero stress

Turbulent viscosity models will not work in this case!





■ exp. Tucker (1970) ● exp. Warhaft (1980)

(from Pope "Turbulent flows")

Assumption of stress/strain alignment

Boussinesq:

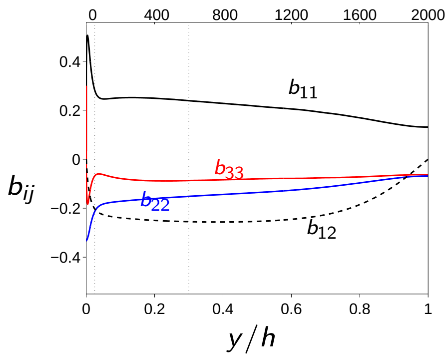
$$b_{ij} = -\frac{\nu_T}{k}\bar{S}_{ij}$$

But, data shows:

- even in simple equilibrium flows

  $\longrightarrow$ anisotropy NOT aligned with mean strain rate
- example: plane channel flow
- problem worse in more complex flows

DNS data for channel flow (cf. lecture 6)



(Jimenez et al., $Re_\tau = 2000$)

The analogy: Newtonian stress/turbulent viscosity

Kinetic theory for ideal gases → Newtonian stress law

$$\boxed{-\sigma_{ij}/\rho - p/\rho\delta_{ij} = -2\nu S_{ij}} \qquad \text{with: } \nu \approx \tfrac{1}{2}\bar{C}\lambda$$

- $\bar{C}$ mean molecular speed, λ mean free path
- time scale ratio in shear flow: $\frac{\lambda}{\bar{C}}\mathcal{S} = \mathcal{O}(10^{-10})$

Eddy viscosity hypothesis for turbulent flow

$$\boxed{\langle u_i' u_j' \rangle - \frac{2}{3} k\, \delta_{ij} = -2\nu_T \bar{S}_{ij}}$$

- typical time scale ratio: $\frac{k}{\varepsilon}\mathcal{S} = \mathcal{O}(1)$
- local equilibrium assumption in general NOT valid!

Linear turbulent viscosity models

How can the turbulent viscosity ν_T be determined?

- uniform turbulent viscosity (cf. lecture 4)
- algebraic expressions (mixing-length etc.)
- one-equation models (k-model, Spalart-Allmaras)
- two-equation models (k-ε, k-ω) (cf. lecture 9)

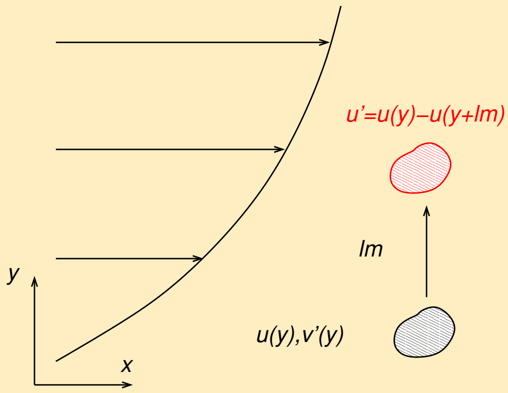
Mixing-length model (Prandtl 1925)

Consider two-dimensional shear flow (channel or BL)

- dimensionally: $\boxed{\nu_T = u^* \cdot \ell_m}$
- fluid "lump" travels $\delta y = \ell_m$
- maintains original $u(y)$
- for constant shear $\mathcal{S}$:
  $u' = -\mathcal{S} \cdot \ell_m$
- Prandtl's approximation:

$$u^* \approx \ell_m \left| \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} \right|$$

$$\Rightarrow \boxed{\nu_T = \ell_m^2 \left| \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} \right|}$$



Mixing-length coefficients for different flows

Self-similar free shear flows

- mixing length: $\ell_m = \alpha \cdot r_{1/2}$

	α
plane wake	0.180
mixing layer	0.071
plane jet	0.098
round jet	0.080

(from Wilcox 2006)

Fully-developed wall-bounded shear flows

- van Driest function for buffer and log-region:

$$\ell_m = \kappa y \left(1 - \exp(-y^+/A^+)\right) \qquad A^+ = 26$$

- simple cut-off for the outer region: $\max(\ell_m) = 0.09\,\delta$
- more elaborate models for boundary layers:
  Cebeci & Smith (1967), Baldwin & Lomax (1978)

Assessment of mixing-length models

Advantage

- numerically efficient:

  only solve averaged Navier-Stokes + algebraic expressions

Drawbacks

- turbulent velocity scale entirely determined by mean flow
- incompleteness: flow-dependent mixing length

Turbulent kinetic energy model

$$\boxed{\langle u_i' u_j' \rangle - \frac{2}{3} k \, \delta_{ij} = -2 \nu_T \bar{S}_{ij}} \qquad \boxed{\nu_T = u^* \cdot \ell^*}$$

Determine characteristic velocity u^* from TKE

- u^* often not given by mean flow

  e.g. decaying grid turbulence
- Kolmogorov (1942), Prandtl (1945):

  $u^* = c\sqrt{k}$ with: $c = 0.55$, and: $\ell^* = \ell_m$

$\Rightarrow$ determine k from transport equation

$\rightsquigarrow$ ℓ_m still needs to be provided flow by flow

Turbulent kinetic energy model: closure

The TKE transport equation (cf. lecture 4)

$$\frac{\bar{D}k}{\bar{D}t} - \mathcal{P} = -\Big(\underbrace{\frac{1}{2}\langle u_i'u_i'u_j'\rangle + \langle u_j'p'\rangle/\rho - \nu k_{,j}}_{\tilde{\mathbf{T}}'}\Big)_{,j} - \tilde{\varepsilon}$$

- production term *closed* through Boussinesq hypothesis
- model for dissipation from high-Re assumption:

  $\tilde{\varepsilon} = C_D\, k^{3/2}/\ell_m$ with: $C_D = c^3$ (from log-law)
- model for flux term from gradient-transport hypothesis:

  $\tilde{\mathbf{T}}' = -\left(\nu + \frac{\nu_T}{\sigma_k}\right)\nabla k$ with: $\sigma_k = 1$

Prediction of the individual model terms (1)

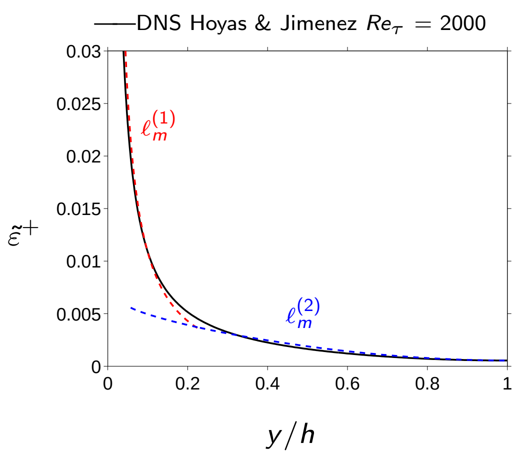
Algebraic dissipation model

- $\tilde{\varepsilon} = C_D\, k^{3/2}/\ell_m$
- consider plane channel flow
- with adapted constant: $C_D = 0.125$
- 2-layer mixing length:

  $\ell_m^{(1)} = \kappa y\,(1 - \exp(-y^+/A^+))$

  $\ell_m^{(2)} = 0.09\,\delta$
- reasonable in outer region

⇝ strong discrepancies near the wall ($y^+ < 40$)

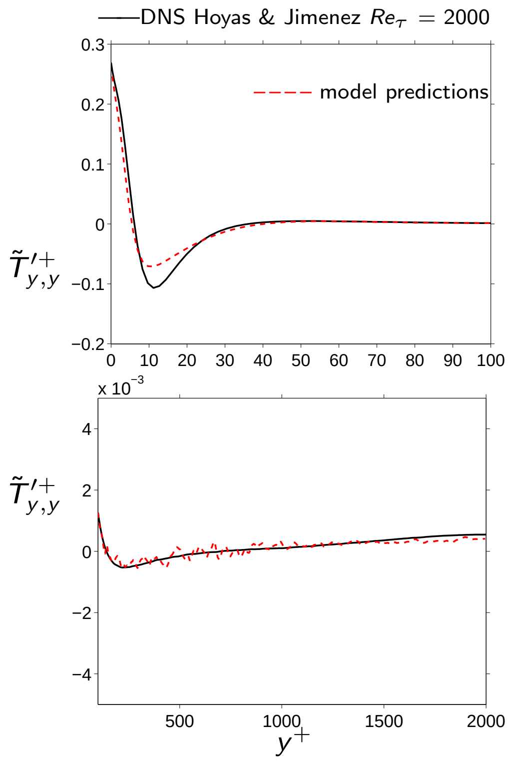


Prediction of the individual model terms (2)

Model for the energy flux

- $\tilde{\mathbf{T}}' = -\left(\nu + \frac{\nu_T}{\sigma_k}\right)\nabla k$
- plane channel flow
- usual value: $\sigma_k = 1$
- reasonable model

⇝ some discrepancies in buffer layer ($10 \leq y^+ \leq 20$)



Incompleteness of the TKE model

Problem of the one-equation model based on TKE

⇝ the lenght scale ℓ^* needs to be specified

⇒ incompleteness

Is there a "complete" one-equation model?

⇒ models with transport equation for turbulent viscosity ν_T

- Nee & Kovasznay (1969)
- Baldwin & Barth (1990)
- Spalart & Allmaras (1992)
- Menter (1994)

The Spalart-Allmaras model for turbulent viscosity

$$\frac{\bar{\mathrm{D}}\nu_T}{\bar{\mathrm{D}}t} = \nabla \cdot \left(\frac{\nu_T}{\sigma_\nu} \nabla \nu_T \right) + S_\nu(\nu, \nu_T, \bar{\mathbf{\Omega}}, |\nabla \nu_T|, \ell_w)$$

- convection-diffusion equation + source term
- source includes various mechanisms of generation/destruction
  - mean flow rotation $\bar{\mathbf{\Omega}}$
  - near-wall behavior through wall-distance ℓ_w
  - destruction term $(|\nabla \nu_T|^2)$, ...
- basic model: 8 closure coefficients, 3 closure functions
- calibrated for aerodynamical applications

Assessment of the Spalart-Allmaras model

Spreading rate of free shear flows

	SA model	measured
plane wake	0.341	0.32-0.40
mixing layer	0.109	0.103-0.120
plane jet	0.157	0.10-0.11
round jet	0.248	0.086-0.096

Skin friction of boundary layers

pressure gradient	SA model error
favorable	1%
mild adverse	10%
moderate adverse	10%
strong adverse	33%

(from Wilcox 2006)

- ⇝ not satisfactory in some free shear flows
- reasonable predictions for attached boundary layers
- ⇝ discrepancies in separated flows

⇒ Need a more universal model for general flows

Summary

Main issues of the present lecture

- How can the Reynolds-averaged equations be closed?
- What are the different types of models commonly used?
  - Boussinesq's turbulent viscosity hypothesis
    - algebraic models
    - transport equations for one or two turbulent scales
  - transport equations for the Reynolds stress
- Do simple eddy viscosity models allow for acceptable predictions?
  - mixing-length type models are not complete
  - one-equations models offer modest advantages
  - ⇝ both types lack universality

Outlook on next lecture: k–ε and other eddy viscosity models

How can the turbulent viscosity be completely determined from field equations?

Does this improve the predictive capability?

Further reading

- S. Pope, *Turbulent flows*, 2000
  $\longrightarrow$ chapter 8 & 10
- P.A. Durbin and B.A. Pettersson Reif, *Statistical theory and modeling for turbulent flows*, 2003
  $\longrightarrow$ chapter 6
- D.C. Wilcox, *Turbulence modeling for CFD*, 2006
  $\longrightarrow$ chapter 2, 3 & 4