Turbulenzmodelle in der Strömungsmechanik

Turbulent flows and their modelling

Markus Uhlmann

Institut für Hydromechanik

www.ifh.uni-karlsruhe.de/people/uhlmann

WS 2008

Summary of last lecture

Lecture 6 – Wall-bounded shear flows

- What is the general structure of wall-bounded flows?
  - inner layer/outer layer: linear/log-law, defect law
- How does the presence of a solid boundary affect the turbulent motion?
  - stronger anisotropy than free shear flows
  - wall has selective effect on velocity components
- What is the effect of wall roughness?
  - shift of the log-law compared to smooth walls

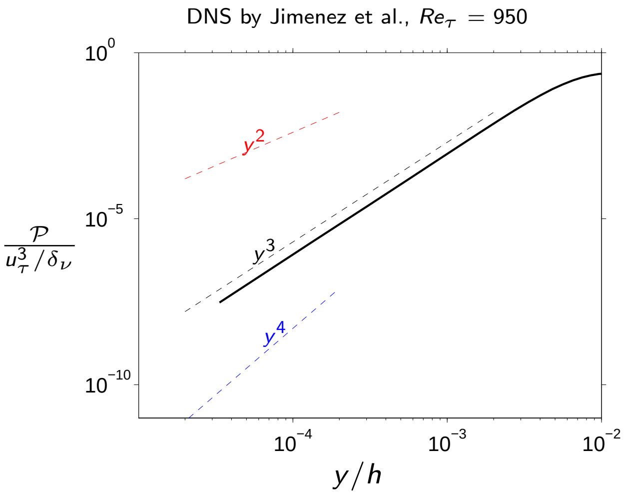
Solution to last week's problem

Determine the variation with wall-distance of the production $\mathcal{P}$ in fully-developed plane channel flow, valid for very small values of y.

Result:

$\mathcal{P}(y) = \mathcal{O}(y^3)$

for $y/h \ll 1$



LECTURE 7

DNS as numerical experiments

Questions to be answered in the present lecture

What are the possibilities & limitations of numerical simulations of the full Navier-Stokes equations?

Part I

- what is DNS?
- why perform DNS?
- what is the history of DNS?
- what are the computational requirements?
- how to treat the boundary conditions?

Part II

- DNS results for coherent structure dynamics in wall flows

Definition of "direct numerical simulation"

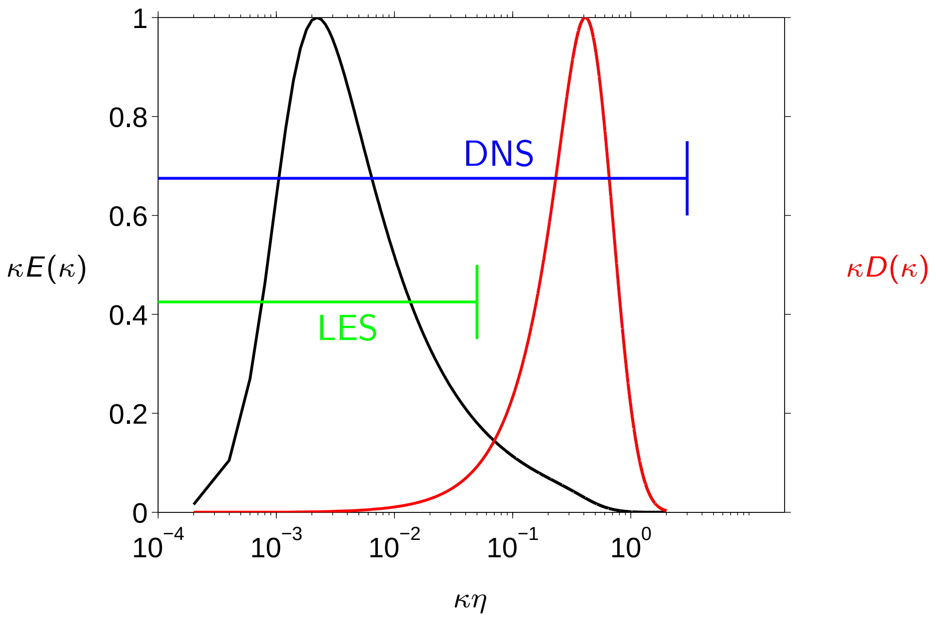
Solve the Navier-Stokes equations for turbulent flow, resolving all relevant temporal and spatial scales.

- for incompressible fluid solve:

$$\begin{aligned} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} + \frac{1}{\rho} \nabla p &= \nu \nabla^2 \mathbf{u} \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

with suitable initial & boundary conditions.

Spectral view: DNS versus LES

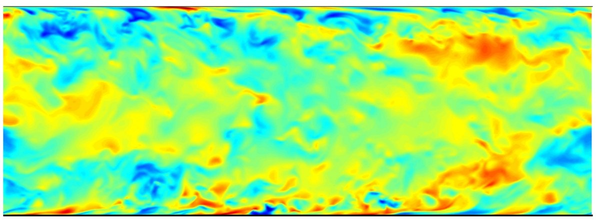


- DNS resolves spatial scales down to Kolmogorov scale η

Physical space view: DNS versus RANS

Example: channel flow

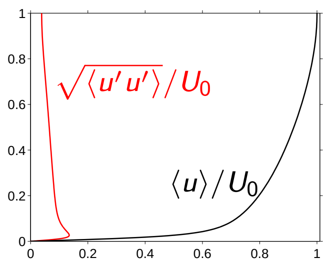
instantaneous DNS data (u')



$\longrightarrow$ flow direction

$\Rightarrow$

DNS statistics



- DNS needs to be integrated in time to obtain statistics
- $\langle u_i \rangle$, $\langle u'_i u'_j \rangle$ are variables in RANS computation

Objectives of DNS studies

(Today) DNS is a research method, not an engineering tool.

- computational effort:

  $\longrightarrow$ today not feasible to perform DNS for practical application

- main purpose of DNS:

  $\longrightarrow$ development of turbulence theory

  $\Rightarrow$ improvement of simplified models

1. DNS as "precise experiment" or "perfect measurement"

If we can simulate the flow with high-fidelity:

- full 3D, time-dependent flow field is available
- virtually any desired quantity can be computed
  (e.g. pressure fluctuations, pressure-deformation tensor)
- there are no limitations by measurement sensitivity
  (e.g. size of probes near a wall)

$\rightsquigarrow$ analysis only limited by mind of researcher
(it is important to ask the right questions)

$\Rightarrow$ DNS complements existing laboratory experiments

2. DNS as "virtual experiment"

When experiments are too costly/impossible to realize:

- numerical simulations provide great flexibility
- idealizations can be realized with ease:
  - e.g. homogeneous-isotropic flow conditions
  - periodicity
  - absence of gravitational force
  - ...

⇒ DNS replaces laboratory experiments

3. DNS as "non-natural experiment"

When non-physical configurations need to be simulated:

- we have the possibility to modify the equations
- we can apply arbitrary constraints
- examples from the past are:
  - filtering (damping) turbulence in some part of the domain
  - suppress individual terms in the equations
  - applying artificial boundary conditions
  - ...

⇒ DNS directly serves turbulence theory

Historical development of DNS

1972 first ever DNS of hom.-iso. turbulence by Orszag & Patterson

1981 homogeneous shear flow by Rogallo

1987 plane channel flow by Kim, Moin & Moser

1986-88 flat-plate boundary layer by Spalart

1990-95 homogeneous compressible flow (Erlebacher/Blaisdell/Sarkar)

1997 particle transport in channel flow (Pan & Banerjee)

2005 deformable bubbles in channel flow (Lu et al.)

currently: wide range of configurations ...

- # of publications in Phys. Fluids: 1990 – 14, 2008 – 76

Numerical requirements for DNS

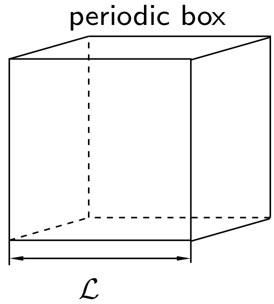
Homogeneous turbulence

- uniform grid with $N \times N \times N$ points: $\Delta x = \Delta y = \Delta z = \frac{\mathcal{L}}{N}$
- assume a periodic field
  $\longrightarrow$ use Fourier series with wavenumbers: $\kappa_i^{(\alpha)} = \frac{2\pi i}{\mathcal{L}}$, where: $-N/2 \le i \le N/2$

$\Rightarrow$ largest wavenumber: $\kappa_{max} = \frac{\pi N}{\mathcal{L}}$

- operation count per time step: using fast Fourier transform $\mathcal{O}(N^3 \log N)$

periodic box



Fourier modes: $\exp(I\boldsymbol{\kappa}\mathbf{x})$

$\boldsymbol{\kappa} = (\kappa^{(1)}, \kappa^{(2)}, \kappa^{(3)})$

Homogeneous turbulence – spatial resolution

Large scale resolution

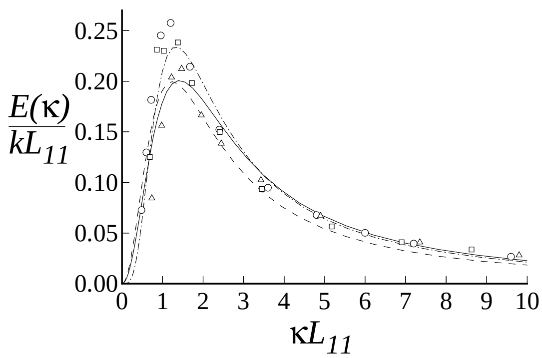
- largest flow scales need to be much smaller than box size
- ⇝ otherwise: artifacts of periodicity!
- rule of thumb: (box) $\mathcal{L} \geq 8L_{11}$ (integral scale)
- recall: largest non-zero wavenumber in DNS is $\kappa_0 = \frac{2\pi}{\mathcal{L}}$
- ⇒ $\kappa_0 L_{11} = \frac{\pi}{4}$

  found to be adequate by comarison with experiments

Homogeneous turbulence – large scale resolution (2)

Energy-containing range

- smallest wavenumber: setting $\kappa_0 L_{11} = \frac{\pi}{4}$
- ⇒ $\approx$ 95% of energy resolved



grid turbulence, Comte-Bellot & Corrsin 1971

∘ $Re_\lambda = 60 \ldots 70$

Homogeneous turbulence – small scale resolution

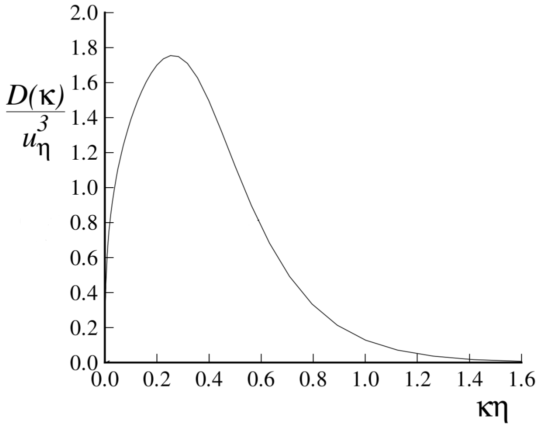
Small scale resolution

- need to resolve the dissipation range
- ⇝ otherwise: there is no sink for kinetic energy → "pile-up"
- rule of thumb: $\kappa_{max}\eta \geq 1.5$ or $\Delta x \leq \frac{\pi\eta}{1.5}$

Homogeneous turbulence – small scale resolution (2)

Dissipation range

- representing up to: $\kappa_{max}\eta = 1.5$
- ⇒ most dissipation resolved



Pope's model spectrum, $Re_\lambda = 600$

Homogeneous turbulence – number of grid points

Combined small/large scale requirements

- $N = \dfrac{\mathcal{L}}{\Delta x} = \dfrac{12 L_{11}}{\pi\eta}$
- how does the scale ratio L_{11}/η evolve with Re?
- from the model spectrum: $L_{11}/L \approx 0.43$ for large Re (recall $L \equiv k^{3/2}/\varepsilon$ from lecture 6)
- defining $Re_L \equiv \frac{k^{1/2}L}{\nu}$ we obtain: $\frac{L}{\eta} = Re_L^{3/4}$

⇒ finally: $\boxed{N \approx 1.6\, Re_L^{3/4}}$ i.e. $\boxed{N^3 \approx 4.4\, Re_L^{9/4}}$

⇝ steep rise with Reynolds!

Homogeneous turbulence – temporal resolution

Resolving the small-scale motion

- typically need: (time step) $\Delta t = 0.1\tau_\eta$ (Kolmogorov scale)

Sampling sufficient large-scale events

- each simulation needs to be run for a time T given by:
  $T \approx 4\,\dfrac{k}{\varepsilon}$ (k/ε is characteristic of large scales)

⇒ obtain for the number of time steps M:

- $\boxed{M = \dfrac{T}{\Delta t} = \dfrac{4}{0.1}\, Re_L^{1/2}}$

Homogeneous turbulence – total operation count

Total number of operations per DNS, using spectral method:

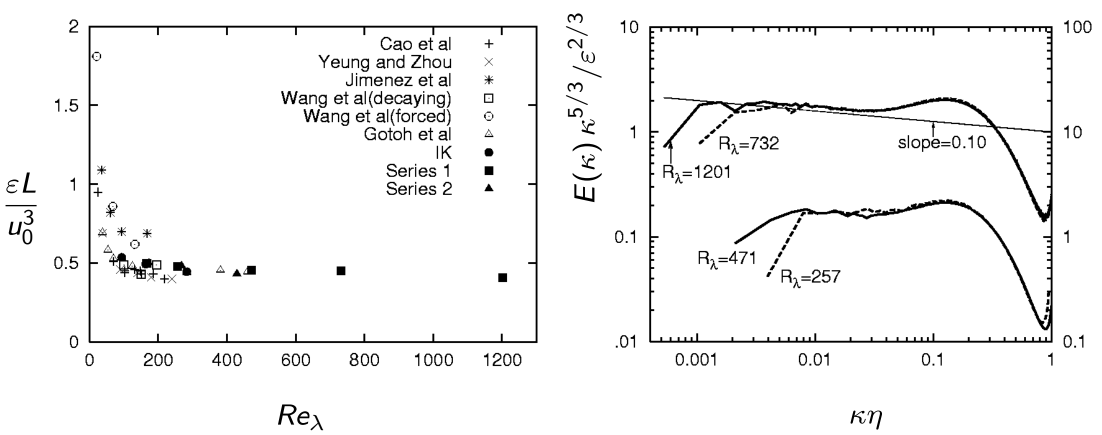
- $N_{tot} = N_{op} \cdot M \sim N^3 \log(N) \cdot M \sim Re_L^{11/4} \log(Re_L)$

Simulation parameters for "landmark" studies:

N	Re_L	computer speed	# processors	
32	180	10 Mflop/s	1	Orszag & Patterson 1972
512	4335	46 Gflop/s	512	Jimenez et al. 1993
4096	216000	16 Tflop/s	4096	Kaneda et al. 2003

Result of high-Reynolds DNS of hom.-iso. turbulence

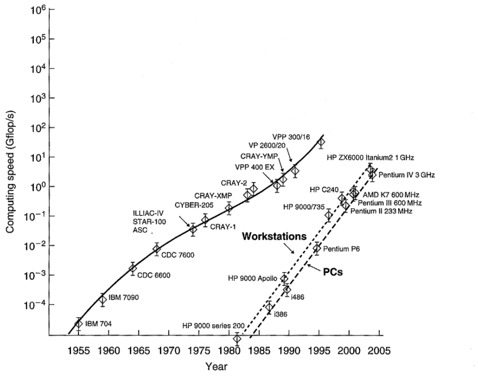
Kolmogorov scaling of data by Kaneda et al. (2003)



- scaling largely confirmed

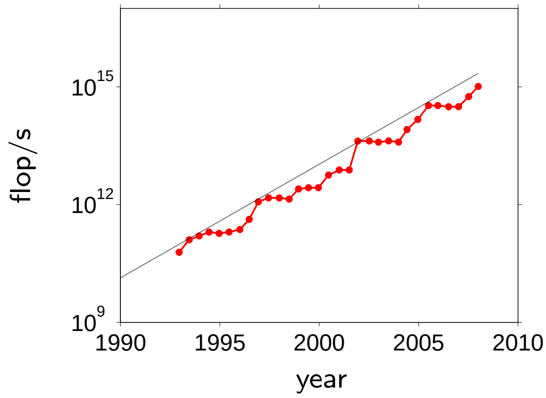
Evolution of computer speed

single-processor CPU speed



(from Hirsch 2007)

performance of multi-processor systems



(data from `top500.org`)

- large CPU speed increase
- limitation: power & heat

- massively-parallel machines maintain exp-growth

$\Rightarrow$ peak performance doubles every 18 months

Boundary conditions for DNS

No particular problems posed by the following boundaries:

- solid walls, homogeneous directions, far-field

The problem of inflow-outflow boundaries:
we need to prescribe turbulence!

1. Taylor's hypothesis $\rightarrow$ temporal instead of spatial variation
2. rescaled outflow used as inflow (Spalart) $\rightarrow$ works for BL
3. impose artificial turbulence at inflow (Le & Moin) $\rightarrow$ long evolution length
4. periodic companion simulation (Na & Moin) $\rightarrow$ generates inflow

Wall turbulence – numerical requirements

Number of grid points, using spectral method:

- $N^3 \approx 0.01\, Re_\tau^3 \left(\frac{L_x}{h}\right) \left(\frac{L_z}{h}\right)$

Total number of operations per DNS, using spectral method:

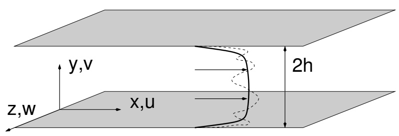
- $N_{tot} \sim Re_\tau^4 \left(\frac{L_x}{h}\right)^2 \left(\frac{L_z}{h}\right)$

Simulation parameters for "landmark" studies:

N^3	Re_τ	L_x/h	L_z/h	
$4 \cdot 10^6$	180	4π	2π	Kim, Moin & Moser 1987
$3.8 \cdot 10^7$	590	2π	π	Moser, Kim & Mansour 1999
$1.8 \cdot 10^{10}$	2000	8π	3π	Hoyas & Jimenez 2006

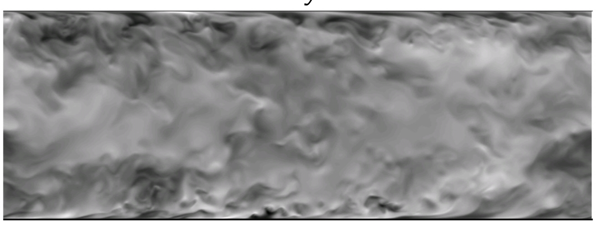
Wall turbulence – visualization

Channel flow at $Re_\tau = 590$



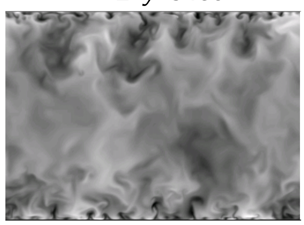
- visualizing streamwise velocity fluctuations u'

x-y slice



$\longrightarrow$ flow direction

z-y slice

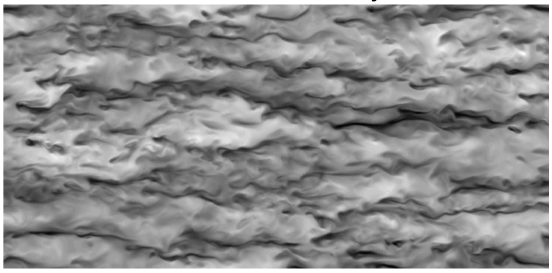


$\otimes$ flow direction

Wall turbulence – visualization (2)

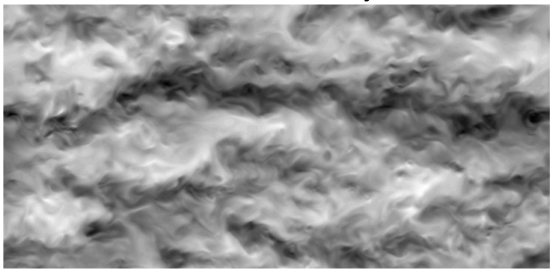
Channel flow at $Re_\tau = 590$, wall-parallel planes, u'

x-z slice, wall-distance $y^+ = 45$



→ flow direction

x-z slice, wall-distance $y^+ = 170$



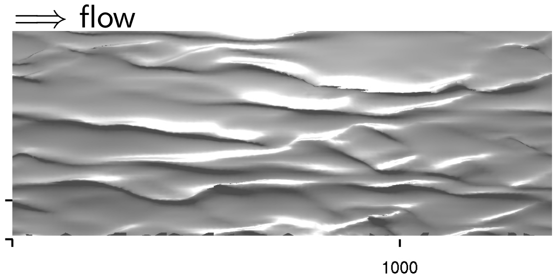
→ flow direction

- typical structures: streamwise velocity "streaks"

→ found in all boundary-layer type flows

Facts about velocity streaks in the buffer layer

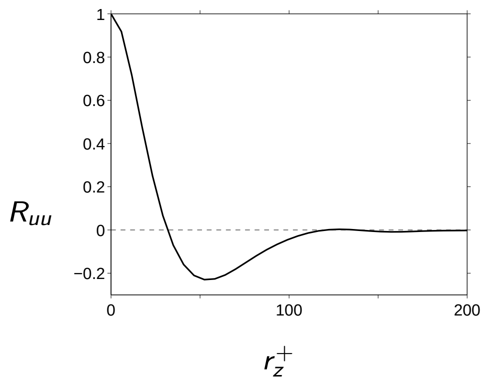
⟹ flow



Statistically speaking:

- lateral spacing of streaks:
  $\Delta\ell_z^+ \approx 100$
- how do we know?

⇒ two-point correlations: minimum of R_{uu} at half of the streak spacing

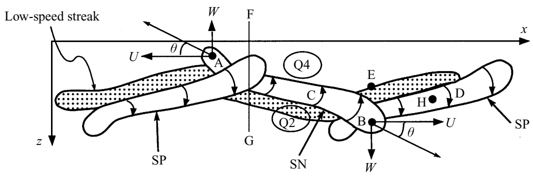


(Moser et al. 1999)

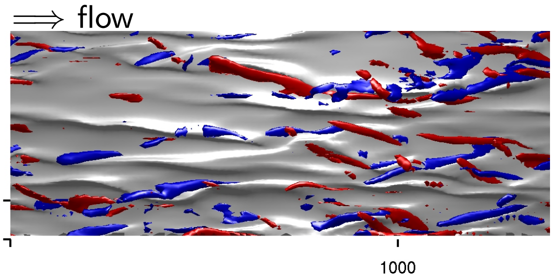
Streamwise vortices

- Streamwise vortices (ω'_x)
  $\ell_x^+ \approx 200$
- associated with streaks

(from Jeong et al. 1997)

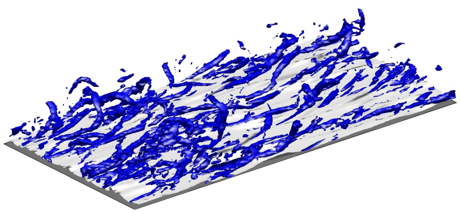


⟹ flow

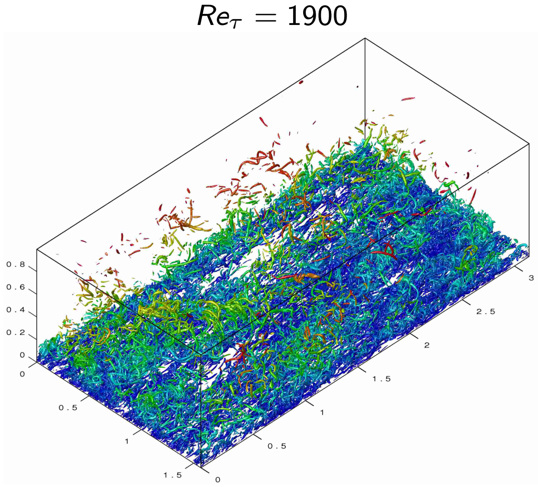


Complex vortex tangles at different Reynolds numbers

$Re_\tau = 180$



$Re_\tau = 1900$

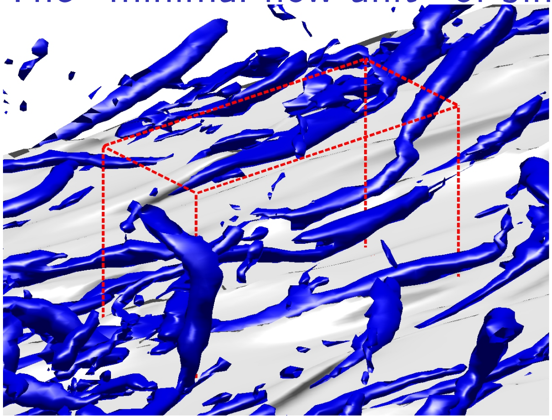


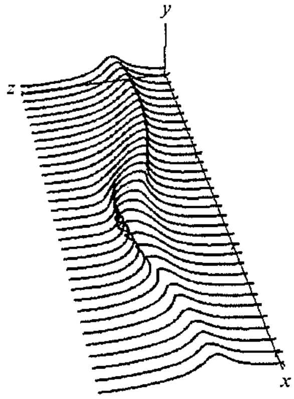
(from del Alamo et al. 2006)

(movie)

Sometimes less is more: reducing the complexity

The "minimal flow unit" of Jimenez & Moin





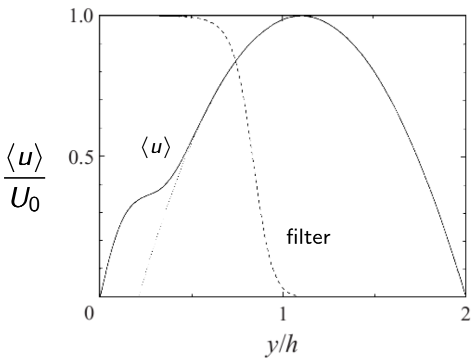
(from Jimenez & Moin 1991)

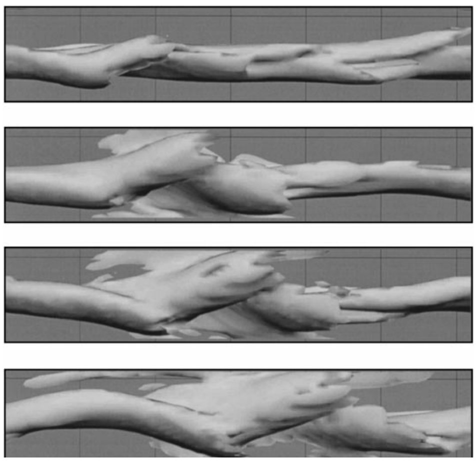
- reducing the box size to a minimum without relaminarizing
- min $L_x^+ \approx 350$, min $L_z^+ \approx 100$

$\Rightarrow$ cheap "laboratory" with principal buffer layer features

Sometimes wrong is right: manipulating the equations

The "autonomous wall" of Jimenez & Pinelli (1999)





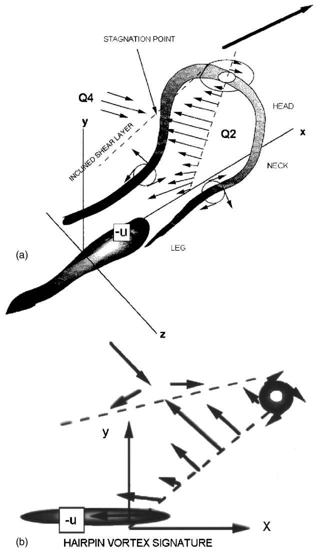
- suppress u' for $y^+ \geq 60$

$\Rightarrow$ turbulence survives!

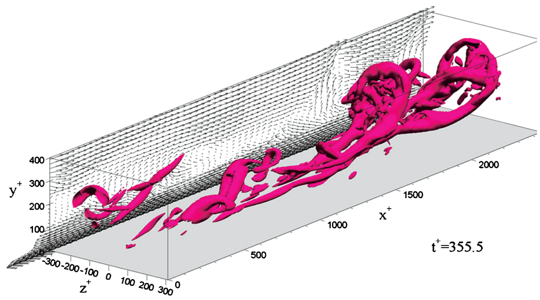
- near-wall region: statistics approximately unchanged
- time sequence of streak break-up (movie)

What happens outside the buffer layer?

Hairpin vortices growing into vortex packets



(sketch from Adrian 2007)



(DNS by Adrian 2007)

- structures in outer region still under investigation!

How can we apply knowledge about coherent structures?

Control of turbulent flow

- "opposition control" (Choi, Moin & Kim 1994)
- imposing $v_{wall}(x,z) = -v'(x, y^+=10, z)$

$\longrightarrow$ up to 25% drag reduction

$\rightsquigarrow$ but: this method is not practical

- other feasible techniques exist, where sensing is performed at the wall

Reduced order models of the wall region

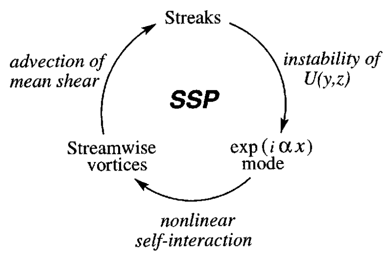
Waleffe's self-sustained process

- generic mechanism
- streamwise vortices generate streaks by advection
- streaks are unstable to sinusoidal perturbations
- perturbations generate new vortices by self-interaction

$\longrightarrow$ 4-equ. model for artificial flow

$\rightsquigarrow$ but: not feasible in practice

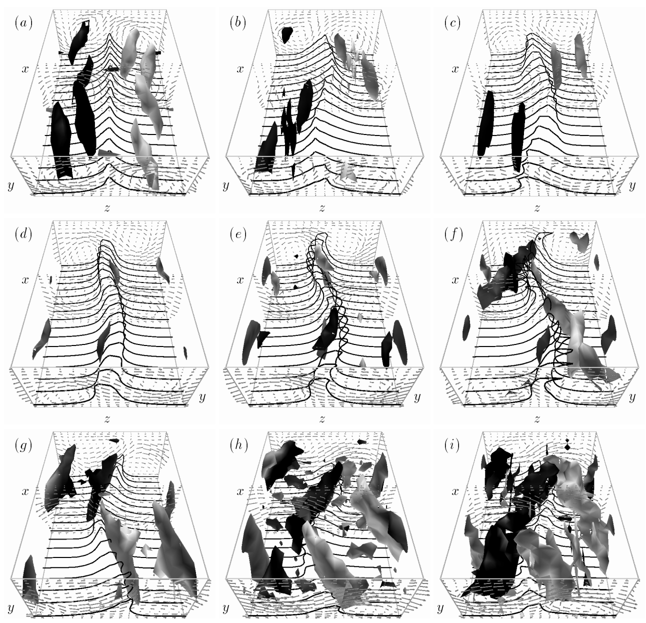
$\Rightarrow$ similar models could be used with LES in future . . .



(from Waleffe 1997)

Periodic solutions: "building blocks" for future models?

Exact periodic solutions are currently pursued in various flows



(movie)

(one period of a Couette flow solution, from Kawahara et al. 2006)

Summary

Main issues of the present lecture

- DNS is useful as a research tool
  - precise experiment/perfect measurement
  - virtual experiment
  - non-natural experiment
- estimates of operation count rise sharply with Reynolds
- suitable inflow boundary conditions are difficult to generate
- streaks & streamwise vortices are fundamental ingredients of turbulence regeneration cycle

Outlook on next lecture: Introduction to RANS modelling

How can the Reynolds-averaged equations be closed?

What are the different types of models commonly used?

Do simple eddy viscosity models allow for acceptable predictions?

Further reading

- S. Pope, *Turbulent flows*, 2000
  $\longrightarrow$ chapter 9 & 7.4
- P. Moin and K. Mahesh, *DNS: A tool in turbulence research*, Annu. Rev. Fluid Mech., 1998, vol 30, pp. 39.
- this is a very active area; more information can be found in the current research literature (Journal of Fluid Mechanics, Physics of Fluids, Journal of Computational Physics)