Turbulenzmodelle in der Strömungsmechanik

Turbulent flows and their modelling

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Summary of last lecture

Lecture 5 – The scales of turbulent motion

- The turbulent energy cascade
  - hierarchy of eddies, downward transfer of energy
  - dissipation determined by large scales, performed by small scales
- Kolmogorov's theory
  - building block of turbulence research
  - valuable results for small scales (e.g. $\kappa^{-5/3}$ spectrum)
- BUT: Problem of non-universality of large scales

LECTURE 6

Wall-bounded shear flows

Questions to be answered in the present lecture

What is the general structure of wall-bounded flows?

How does the presence of a solid boundary affect the turbulent motion?

What is the effect of wall roughness?

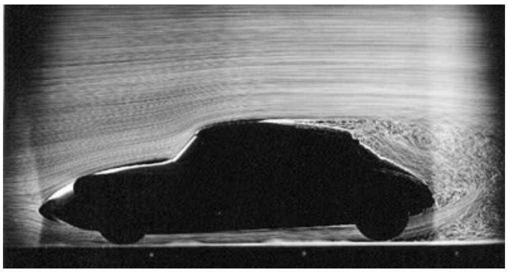
Examples of tubulent wall-bounded flows (1)

Rivers

also: man-made canals, some ocean currents

Examples of tubulent wall-bounded flows (2)

Vehicle aerodynamics



Citroen DS, ONERA

NASA aerodynamic truck

Examples of tubulent wall-bounded flows (3)

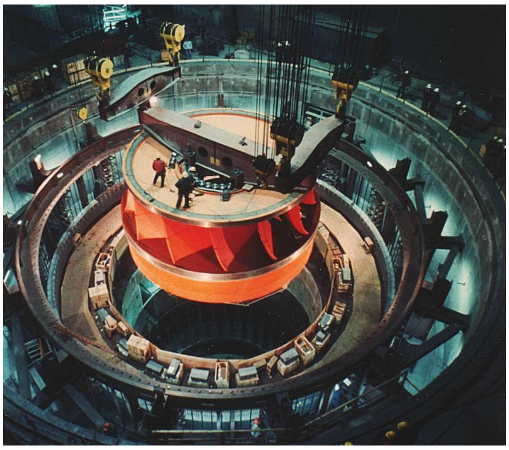
Atmospheric boundary layer:

storm clouds

Examples of tubulent wall-bounded flows (4)

Internal flows:

pipeline systems



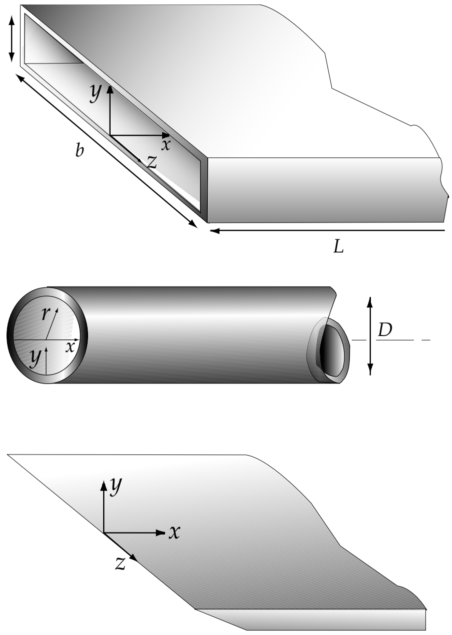
water turbine

also: internal combustion engines, compressors, pumps, blood flow, . . .

Canonical wall-bounded flows

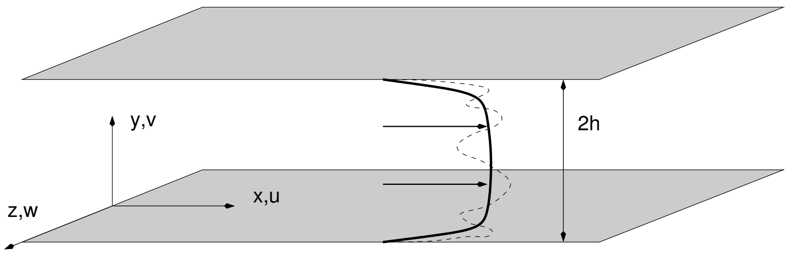
Idealized configurations

- plane channel flow
- pipe flow
- flat-plate boundary layer



Pope "Turbulence"

Plane channel flow configuration



- statistically stationary & fully developed (far from inlet)

$\longrightarrow$ homogeneous in x- & z-direction

- definition of "bulk velocity": $u_b \equiv \int_0^{2h} \langle u \rangle dy/(2h)$

$\longrightarrow$ bulk Reynolds number: $Re_b \equiv u_b h/\nu$

- Reynolds at centerline: $Re_0 \equiv U_0 h/\nu$ with $U_0 = \langle u \rangle (y = h)$

Mean flow equations

Continuity equation (considering x-z-homogeneity)

$$\cancel{\partial_x \langle u \rangle} + \partial_y \langle v \rangle + \cancel{\partial_z \langle w \rangle} = 0$$

$\Rightarrow$ mean wall-normal velocity $\langle v \rangle(y) = 0$, since $\langle v \rangle = 0$ at walls

Mean wall-normal momentum equation

$$\frac{1}{\rho}\partial_y \langle p \rangle + \partial_y \langle v'v' \rangle = 0$$

$\Rightarrow$ mean axial pressure gradient is constant: $\partial_x \langle p \rangle = cst.$

Mean flow equations (2)

Mean streamwise momentum equation

$$\frac{1}{\rho}\partial_x \langle p \rangle + \partial_y \langle u'v' \rangle = \nu \partial_{yy} \langle u \rangle$$

- define total shear stress: $\tau \equiv \rho\nu\partial_y \langle u \rangle - \rho \langle u'v' \rangle$

$\Rightarrow$ varies linearly: $\boxed{\tau(y) = \tau_w(1 - \frac{y}{h})}$ where $\tau_w \equiv \tau(y = 0)$

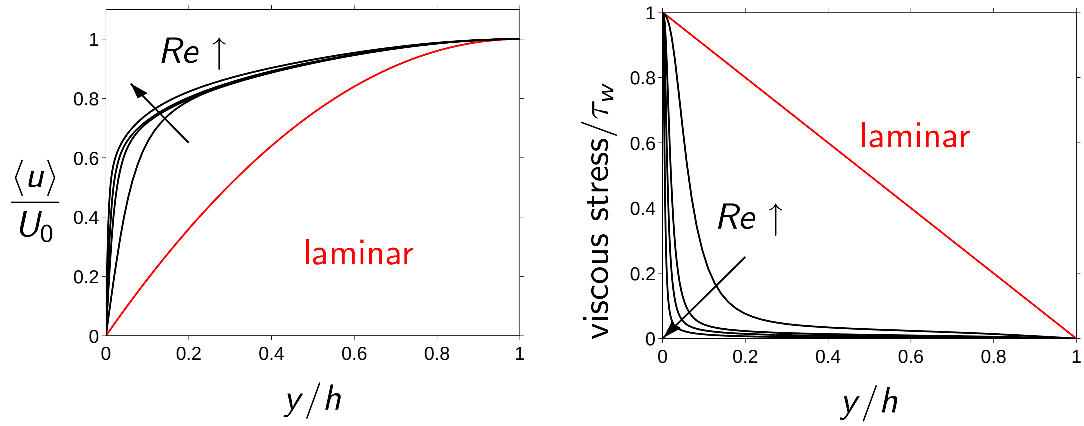
- integral balance yields: $\partial_x \langle p \rangle = -\tau_w / h$

In <u>laminar</u> flow we obtain:

- $\langle u \rangle(y) = \frac{\tau_w}{\rho\nu}\frac{y}{2}\left(2 - \frac{y}{h}\right)$, $u_b = \frac{2}{3}U_0$
- friction factor: $c_f \equiv \frac{\tau_w}{\rho u_b^2/2} = \frac{6}{Re_b}$

Mean velocity profile

DNS data of Jimenez et al.



- mean velocity profiles depend upon Reynolds number (contrary to free shear flows!)
- viscous stress $\rho\nu\partial_y\langle u\rangle$ is *always* important near the wall

Characteristic scales in the near-wall region

Viscosity and wall shear-stress are important parameters

- define friction velocity and viscous length scale:

$$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}\,, \qquad \delta_\nu \equiv \nu\sqrt{\frac{\rho}{\tau_w}} = \frac{\nu}{u_\tau}$$

$\longrightarrow$ friction Reynolds number: $Re_\tau \equiv \dfrac{u_\tau h}{\nu} = \dfrac{h}{\delta_\nu}$

- wall-distance in viscous lengths ("wall units"):

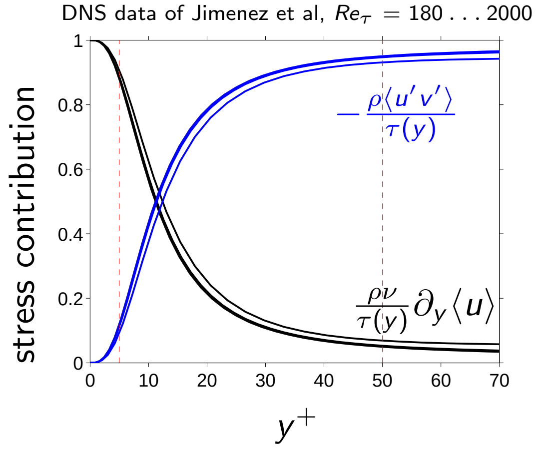
$$y^+ \equiv \frac{y}{\delta_\nu} = \frac{u_\tau y}{\nu}$$

$\longrightarrow$ y^+ equivalent to a local Reynolds number

Importance of viscous and turbulent effects

Data shows near the wall:

- stress curves collapse when plotted against y^+
- viscous contribution decreases with y^+



- $y^+ < 5$: viscous sublayer – Reynolds stress negligible
- $y^+ < 50$: viscous wall-region – direct effect of viscosity
- $y^+ > 50$: outer layer – no direct viscous effect

Scaling of the mean velocity profile

Selecting the relevant parameters – dimensional considerations

- channel flow is fully determined by: ρ, ν, h, u_τ
- wall-distance y is the only relevant coordinate

$\longrightarrow$ two non-dimensional numbers (y/h & Re_τ or y/h & y^+)

$\Rightarrow$ we can write the ansatz: $\boxed{\frac{\mathrm{d}\langle u\rangle}{\mathrm{d}y} = \frac{u_\tau}{y} \cdot \Phi\left(y^+, \frac{y}{h}\right)}$

- Φ is a general non-dimensional function

The inner layer – the law of the wall

For $y/h \ll 1$ velocity profile is independent of h

- $\lim\limits_{y/h \to 0} \Phi\left(y^+, \frac{y}{h}\right) = \Phi_I\left(y^+\right)$

$\rightarrow$ this yields: $\dfrac{du^+}{dy^+} = \dfrac{1}{y^+} \cdot \Phi_I\left(y^+\right)$, with: $u^+ \equiv \langle u \rangle / u_\tau$

- integration gives: $\boxed{u^+ = f_w(y^+)}$ "law of the wall"
- experiments show:

  $f_w(y^+)$ is universal function (BL, channel, pipe ...)

Mean flow behavior derived from the law of the wall

Viscous sublayer ($y^+ < 5$)

- Taylor expansion around $y=0$:

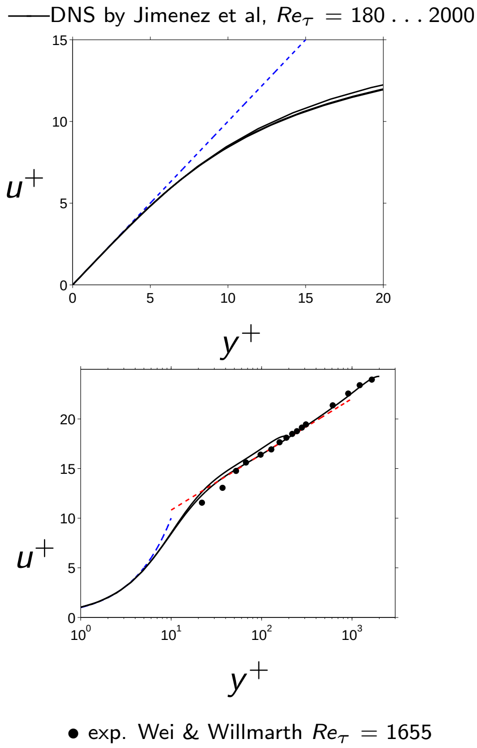
  $\boxed{u^+ = y^+}$ "linear law"

Logarithmic layer ($y^+ > 30$)

- for large y^+, the function $\Phi_I(y^+)$ becomes constant:

  $\boxed{u^+ = \dfrac{1}{\kappa}\log(y^+) + B}$ "log-law"

- $\kappa \approx 0.41,\ B \approx 5.2$



Outer layer – velocity defect law

For $y^+ > 50$: velocity profile independent of viscosity

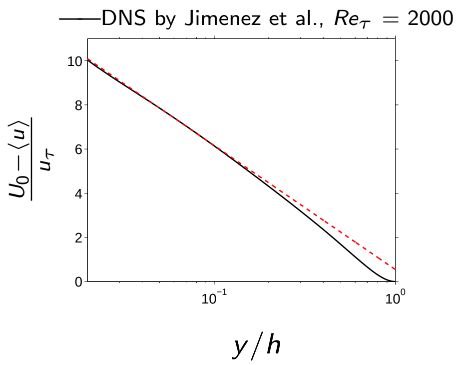
- $\lim_{y/h \to \infty} \Phi\left(y^+, \frac{y}{h}\right) = \Phi_O\left(\frac{y}{h}\right)$

$\rightarrow$ this yields: $\frac{du}{dy} = \frac{u_\tau}{y} \cdot \Phi_O\left(\frac{y}{h}\right)$

- integration from y to h:

$$\boxed{\frac{U_0 - \langle u \rangle}{u_\tau} = F_D\left(\frac{y}{h}\right)}$$

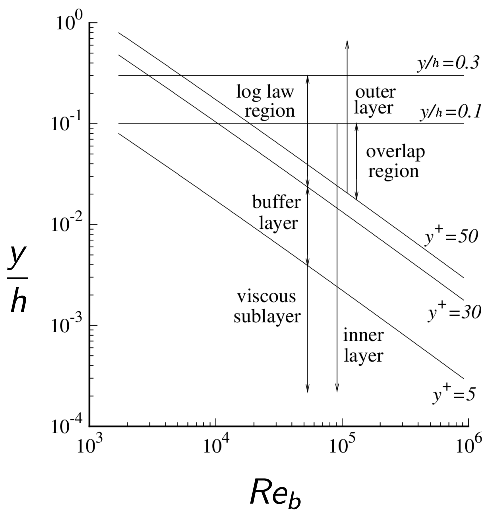
- note: function F_D is not universal!
(different in boundary layer)



for comparison: ----log-law

Summary of regions in near-wall flow

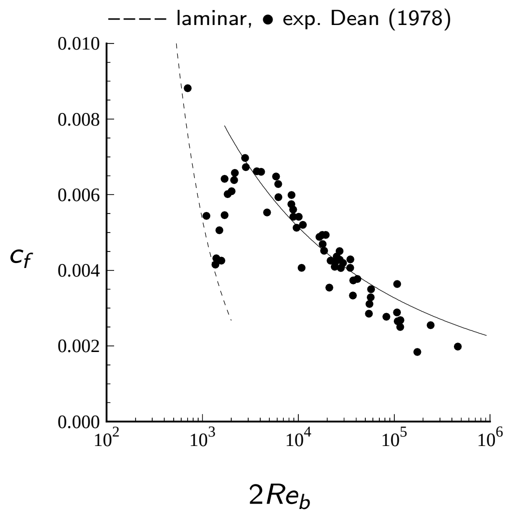
- inner layer: $\langle u \rangle$ independent of U_0 and h
- viscous wall region: viscous contribution of shear stress significant ($y^+ < 50$)
- viscous sublayer: Reynolds shear stress negligible
- buffer layer: region between viscous sublayer and log-region
- logarithmic region: the log-law holds
- outer layer: direct effects of viscosity on $\langle u \rangle$ negligible
- overlap region: overlap between inner and outer layers



(from Pope "Turbulence")

The friction law for channel flow

- friction factor:
  $c_f \equiv \frac{\tau_w}{\rho u_b^2/2} = \frac{2u_\tau^2}{u_b^2}$
- transition occurs around $Re_b = 1000$
- integrating log-law:
  $Re_\tau \approx 0.166\, Re_b^{0.88}$



- lengthscale ratio $h/\delta_\nu = Re_\tau$ increases almost linearly with Re_b

$\longrightarrow$ very small viscous scales at high Reynolds numbers

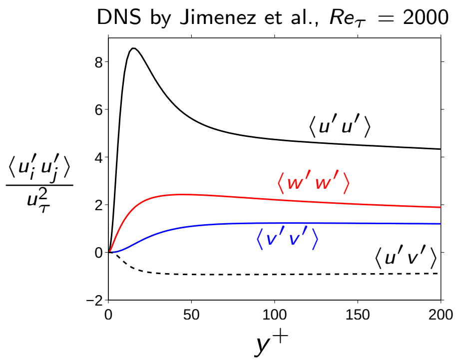
Profiles of the Reynolds stress components

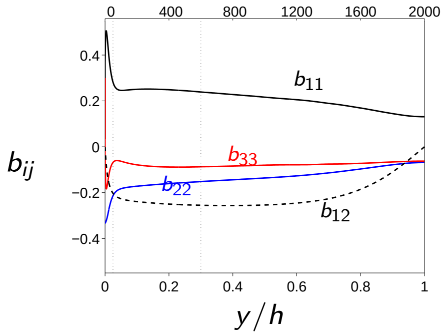
Observations from exp. & DNS

- peak TKE in buffer layer

$\longrightarrow$ signatures of coherent structures (cf. lecture 7)

- anisotropy b_{ij}
  - strong changes in viscous wall region
  - plateau in log-region
  - towards isotropy in core flow





Near-wall asymptotics of velocity fluctuations

Taylor expansion for small wall-distance y:

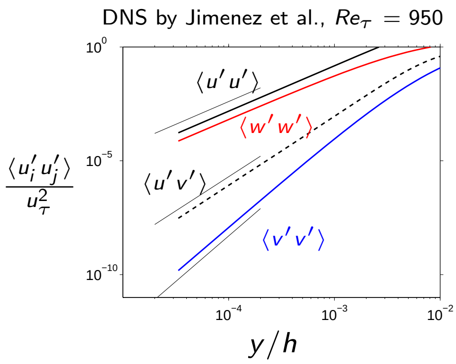
$$
\begin{aligned}
u'(y) &= \cancel{a_1} + b_1\, y + c_1\, y^2 + \ldots \\
v'(y) &= \cancel{a_2} + \cancel{b_2}\, y + c_2\, y^2 + \ldots \\
w'(y) &= \cancel{a_3} + b_3\, y + c_3\, y^2 + \ldots
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\langle u'u' \rangle &= \langle b_1^2 \rangle\, y^2 + \ldots \\
\langle v'v' \rangle &= \langle c_2^2 \rangle\, y^4 + \ldots \\
\langle w'w' \rangle &= \langle b_3^2 \rangle\, y^2 + \ldots \\
\langle u'v' \rangle &= \langle b_1 c_2 \rangle\, y^3 + \ldots
\end{aligned}
$$

- no-slip, impermeability: $a_1 = a_2 = a_3 = 0$
- continuity: $\partial_y v'(y=0) = b_2 = 0$
- taking mean of products of Taylor series:

$\Rightarrow$ Reynolds-stress components have different near-wall slopes $\mathcal{O}(y^2)$, $\mathcal{O}(y^3)$, $\mathcal{O}(y^4)$

Near-wall asymptotics of Reynolds-stresses

- $\langle u'u' \rangle = \mathcal{O}(y^2)$
- $\langle v'v' \rangle = \mathcal{O}(y^4)$
- $\langle w'w' \rangle = \mathcal{O}(y^2)$
- $\langle u'v' \rangle = \mathcal{O}(y^3)$



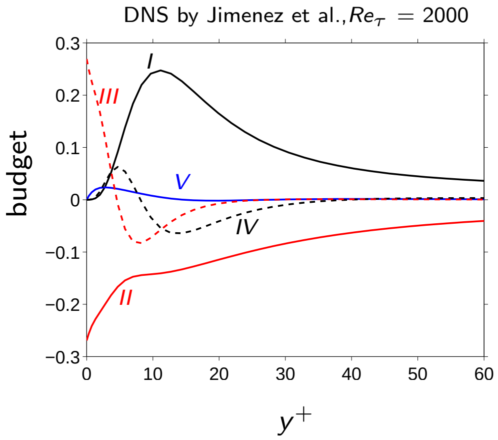
- this knowledge can be used for building consistent models

Turbulent kinetic energy bugdet

Transport equation for statistically stationary channel flow

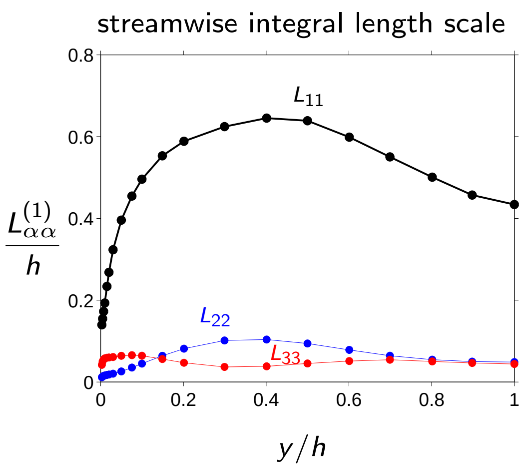
$$\underset{I}{\mathcal{P}} \quad \underset{II}{-\tilde{\varepsilon}} \quad \underset{III}{+\nu\frac{\mathrm{d}^2 k}{\mathrm{d}y^2}} \quad \underset{IV}{-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}y}\langle v' u_j' u_j'\rangle} \quad \underset{V}{-\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}y}\langle v' p'\rangle} \quad = 0$$

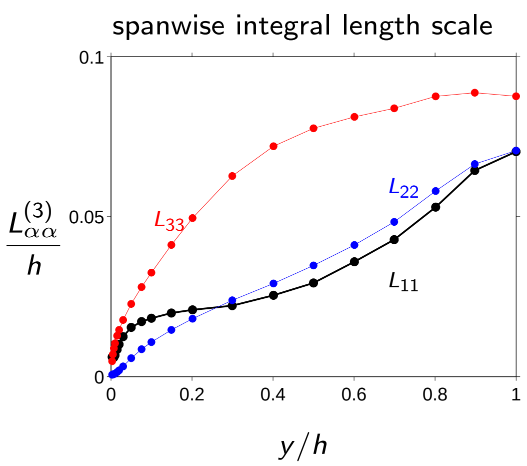
- pseudo-dissipation
  $\tilde{\varepsilon} \equiv \nu\langle u'_{i,j} u'_{i,j}\rangle$
- peak production occurs in buffer layer
- buffer layer exports energy
- dissipation not zero at wall, balanced by visc. diffusion



Length scales in plane channel flow

Observations from DNS by Hoyas & Jimenez ($Re_\tau = 2000$)





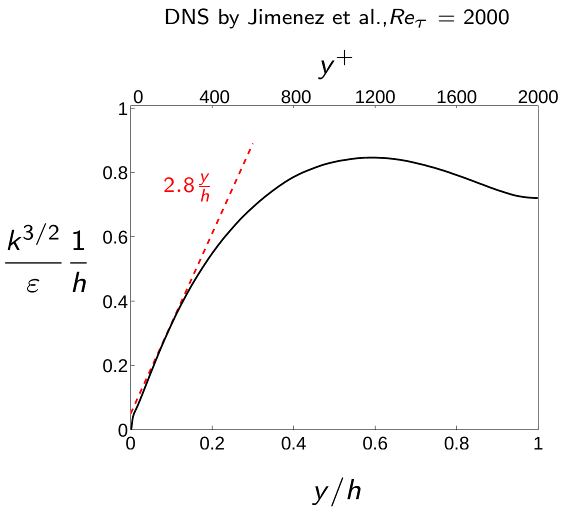
- integral scale depends strongly on component and direction

$\longrightarrow$ turbulence structure highly anisotropic and inhomogeneous

Length scales in plane channel flow (2)

Definition of isotropic length scale

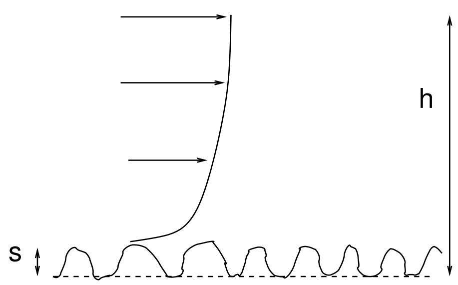
- $L \equiv \dfrac{k^{3/2}}{\varepsilon}$
- no direct physical interpretation
- but: often used in turbulence models
- slope in log-layer: $L/h \sim 2.8y/h$
- in outer region: $L/h \approx 0.8$



Flow over rough walls

- s is characteristic length of protrusions
- roughness: $s \lesssim h/20$ (else: "obstacles")
- details of flow in cavities is complicated
- consequence: ansatz for mean velocity modified

$$\boxed{\frac{d\langle u\rangle}{dy} = \frac{u_\tau}{y} \cdot \bar{\Phi}\left(y^+, \frac{y}{h}, s^+\right)}$$



Logarithmic law for rough walls

Inner layer ($y/h < 0.1$):

$$\lim_{y/h \to 0} \bar{\Phi}\left(y^+, \frac{y}{h}, s^+\right) = \Phi_R\left(y^+, s^+\right)$$

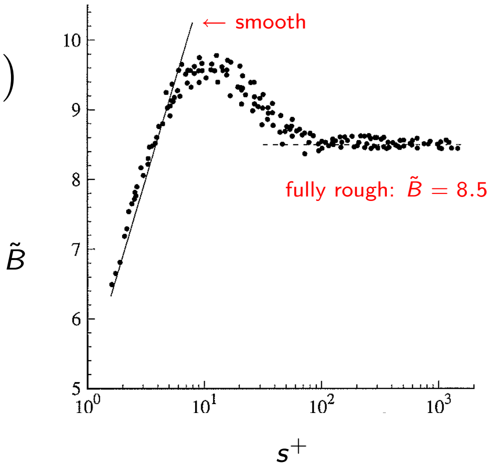
- integration for $y^+ \gg 1$:

$$\boxed{u^+ = \frac{1}{\kappa} \log\left(\frac{y}{s}\right) + \tilde{B}(s^+)}$$

$\Rightarrow$ $\kappa = 0.41$ as for smooth wall

$\Rightarrow$ $\tilde{B}$ depends on s^+

- "fully rough" for $s^+ \gtrsim 70$



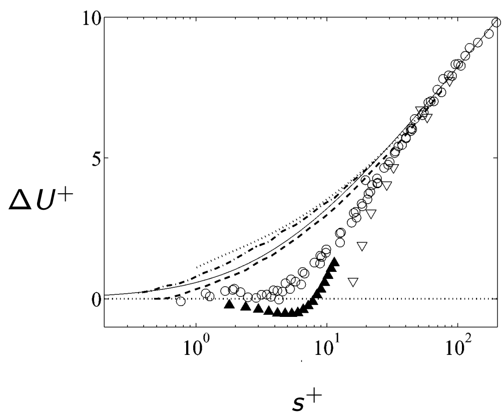
(Nikuradse's data, from Pope "Turbulence")

Transitional roughness – influence of shape

Roughness function ΔU^+

- equivalent form of log-law:
  $u^+ = \frac{1}{\kappa} \log(y^+) + B - \Delta U^+$
- with:
  $\Delta U^+ = \frac{1}{\kappa} \log(s^+) + B - \tilde{B}$
- ΔU^+ measures departure from smooth-wall behavior
- measurements show: friction can be decreased!

(from Jimenez, Ann. Rev. Fluid Mech. 2004)



○	Nikuradse	sand grains
▽	Ligrani & Moffat	spheres
▲	Bechert et al.	riblets

Friction law for rough pipes

Considering pipe flow

- friction factor:

$$c_f \equiv \frac{\tau_w}{\rho u_b^2/2} = \frac{2u_\tau^2}{u_b^2}$$

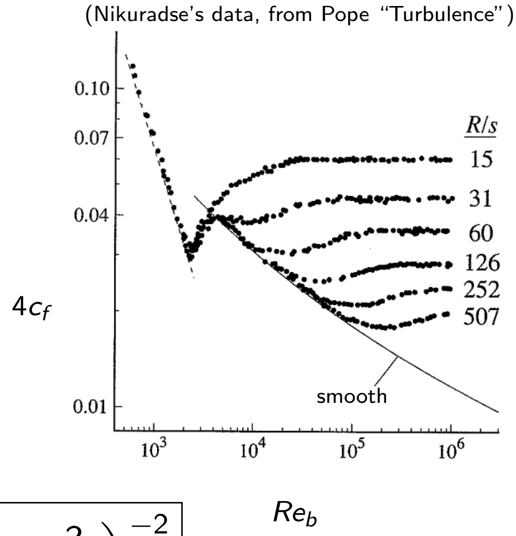
- using velocity defect law:

$(U_0 - u_b)/u_\tau = 3/(2\kappa)$

- using rough-wall log-law:

$$\boxed{c_f = 2\left(\frac{1}{\kappa}\log(R/s) + \tilde{B}(s^+) - \frac{3}{2\kappa}\right)^{-2}}$$

(Nikuradse's data, from Pope "Turbulence")



Summary

Main questions of the present lecture

- What is the general structure of wall-bounded flows?
  - inner layer/outer layer: linear/log-law, defect law
- How does the presence of a solid boundary affect the turbulent motion?
  - stronger anisotropy than free shear flows
  - wall has selective effect on velocity components
- What is the effect of wall roughness?
  - shift of the log-law compared to smooth walls

Discussion of coherent structures → lecture 7

Additional effects on wall-bounded flows

Important effects which are not treated here:

- imposed streamwise pressure gradient
- surface curvature
- system rotation
- geometrical complexities (secondary flows,. . .)

Problem to be solved:

Determine the variation with wall-distance of the production $\mathcal{P}$ in channel flow, valid for very small values of y.

Outlook on next lecture: DNS as numerical experiments

What are the possibilities & limitations of numerical simulations of the full Navier-Stokes equations?

Coherent structures in wall flows

Further reading

- S. Pope, *Turbulent flows*, 2000
  $\longrightarrow$ chapter 7
- H. Schlichting and K. Gersten, *Grenzschicht–Theorie*, 10th edition, 2006
  $\longrightarrow$ part D