Turbulenzmodelle in der Strömungsmechanik

Turbulent flows and their modelling

Markus Uhlmann

Institut für Hydromechanik

www.ifh.uni-karlsruhe.de/people/uhlmann

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Schedule

1	General introduction to turbulent flows	20.10.
2	Equations of fluid motion	27.10.
3	Statistical description of turbulence	10.11.
4	Free shear flows	17.11.
5	The scales of turbulent motion	24.11.
6	Wall-bounded shear flows	1.12.
7	DNS as numerical experiments	8.12.
8	Introduction to RANS modelling	15.12.
9	k–ε and other eddy viscosity models	12.1.
10	Reynolds-stress transport models	19.1.
11	Boundary conditions and wall treatment	26.1.
12	Algebraic stress models	2.2.

General outline of the course

Questions to be discussed:

- How do turbulent flows behave?
- How can turbulence be described quantitatively?
- What are the fundamental physical processes involved?
- How can we model or simulate those flows?

Introduction to turbulence
Conclusion

Literature

- S. Pope, *Turbulent flows*, 2000
  → this lecture roughly follows Pope's book

LECTURE 1:

Introduction to turbulence

Questions to be answered in the present lecture

Why is fluid turbulence important?

What are the characteristics of turbulence?

What are the principal difficulties in describing turbulence?

We are surrounded by turbulence

Turbulence can easily be observed in:

- mixing milk into cup of coffee
- wind motion in the atmosphere
- exhaust from chimneys, cigarettes
- flames
- rivers, waterfalls, ocean currents
- boundary layers on aircraft wings, jet engine exhaust
- ...

Visualizing turbulent flow

Flow structures can be made visible by introducing:

- smoke
- colored dye
- hydrogen bubbles
- tracer particles

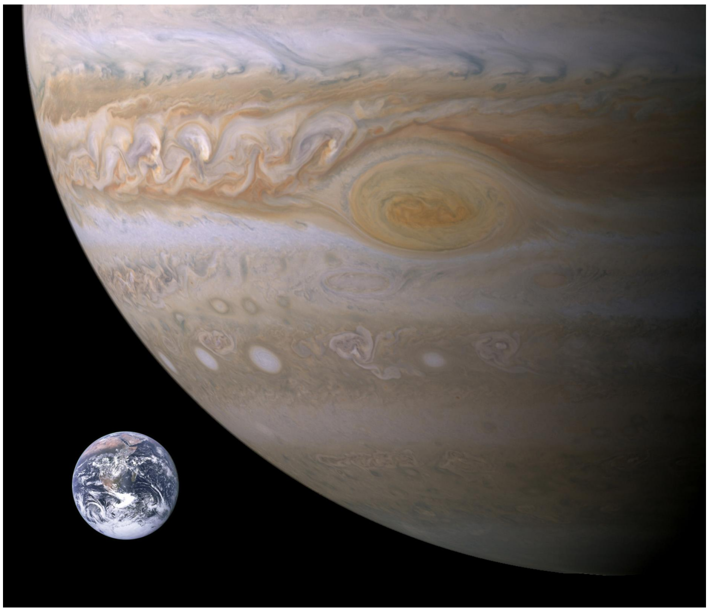
Also: directly visualize density fluctuations of the fluid

- Schlieren, shadowgraph

Examples of turbulent flows (1)

Atmospheric boundary layers

- Earth, Jupiter



Examples of turbulent flows (2)

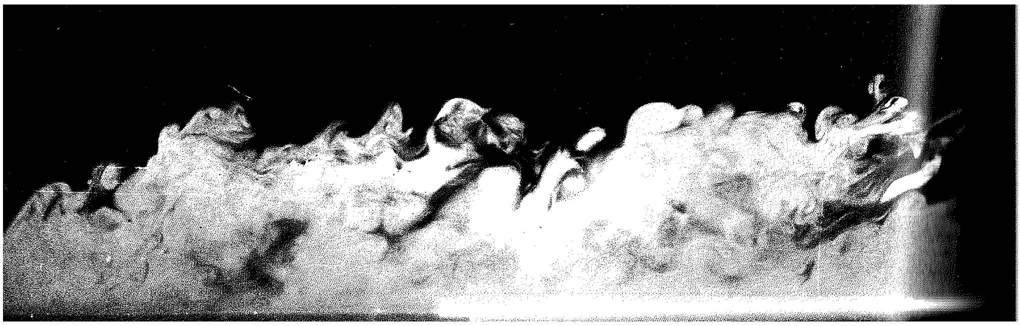
Open channel flow over gravel bed

- $Re \approx 2 \cdot 10^5$, (M. Detert 2008)

$\Rightarrow$ turbulent flow is unsteady, complex patterns

Examples of turbulent flows (3)

Boundary layer, $Re = 4000$

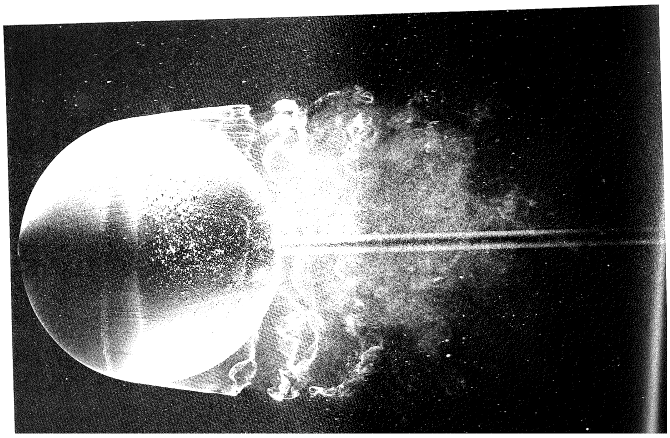


$\longrightarrow$ flow direction

Examples of turbulent flows (4)

Flow around a sphere

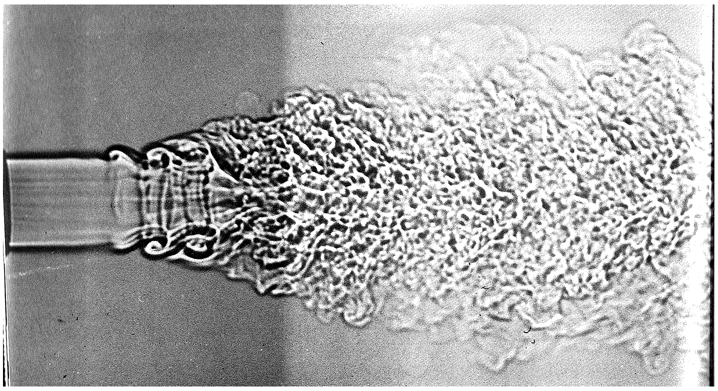
- $Re = 15000$
  turbulent



Examples of turbulent flows (5)

Jet flow

- $Re = 3000$

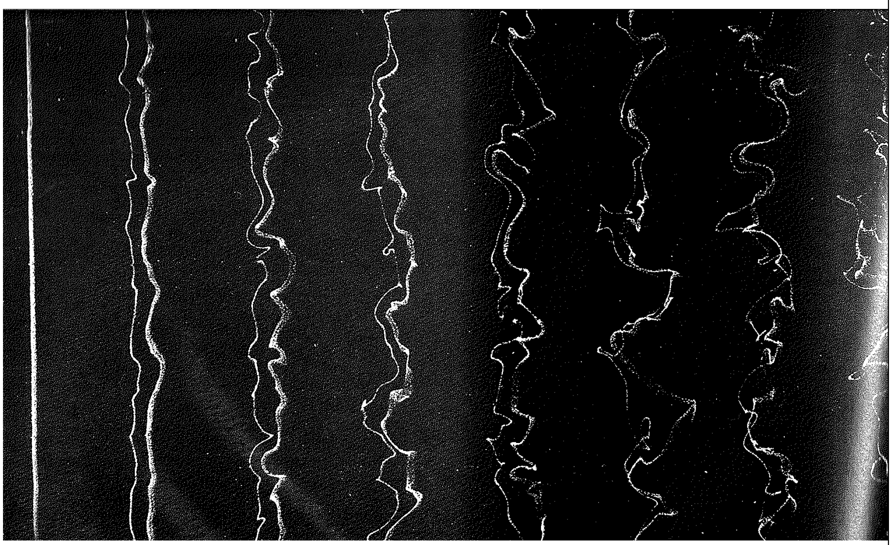


laminar $\longrightarrow$ transitional $\longrightarrow$ turbulent

Examples of turbulent flows (6)

Grid-generated turbulence, nearly isotropic

- $Re = 1360$ (lines of H-bubbles)



Turbulent flows are ubiquitous

Majority of flows are turbulent in:

- geophysics (atmosphere, oceans, rivers)
- engineering (vehicles, combustion, pipes, ...)

Features evident from visualization:

- there is a critical parameter value for turbulence to arise
- unsteady, complex vortical patterns, 3D
- fine scales

Why does the engineer care about turbulent flow?

Design: requires description/prediction of

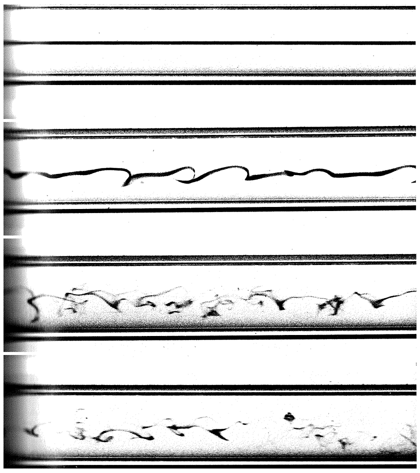
- hydrodynamic forces
  - vehicles, buildings, pipes ...
- heat transfer
  - combustion chambers, ocean surface, nuclear plants ...
- mixing
  - combustion processes, waste water treatment, chemical engineering ...
- acoustics
  - noise generation by vehicles, engines ...

$\Rightarrow$ strongly influenced by turbulence!

There is a critical value of the Reynolds number

Example: Reynold's pipe flow experiment

Re increases ↓



$$Re = \frac{\text{inertial forces}}{\text{viscous forces}}$$

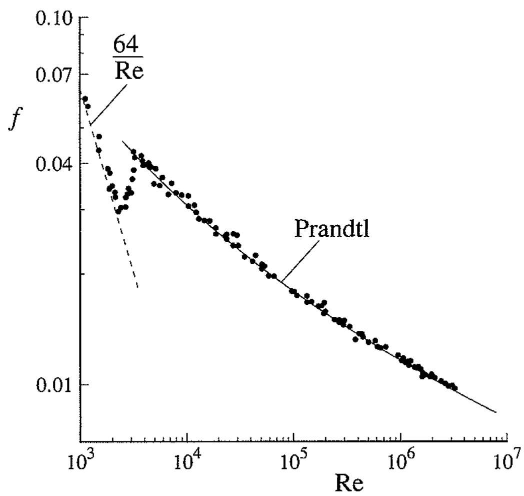
$$Re = \frac{U\,D}{\nu}$$

$$Re_{crit} \approx 2000$$

Consequences of turbulence

Increased momentum transfer: higher wall friction

friction factor

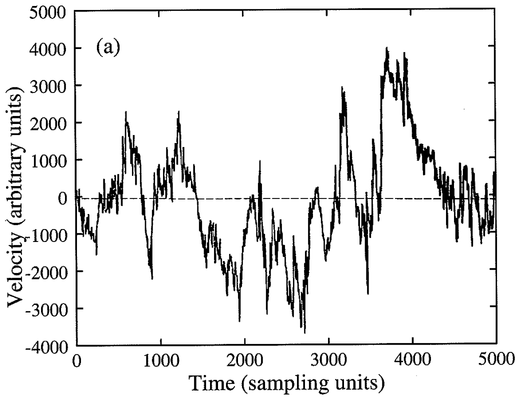


Reynolds number

The random nature of turbulence

Time series of flow velocity

a turbulent signal

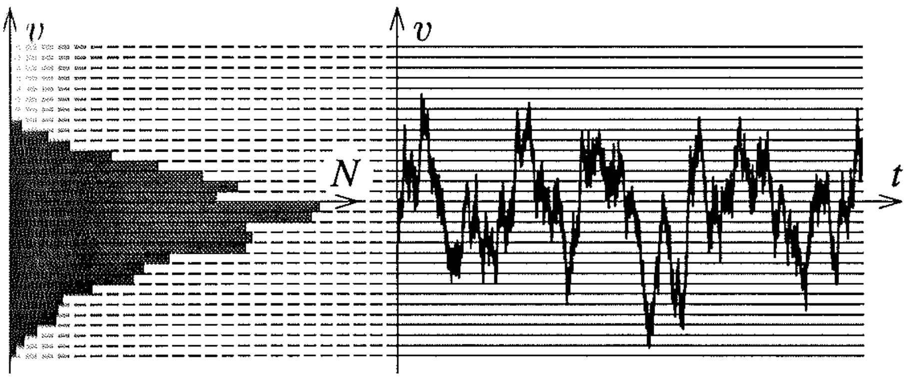


- signals are disorganized, many scales
- unpredictable in detail
- some reproducible properties

(Frisch “Turbulence”, 1995)

The random nature of turbulence (2)

Computing a histogram

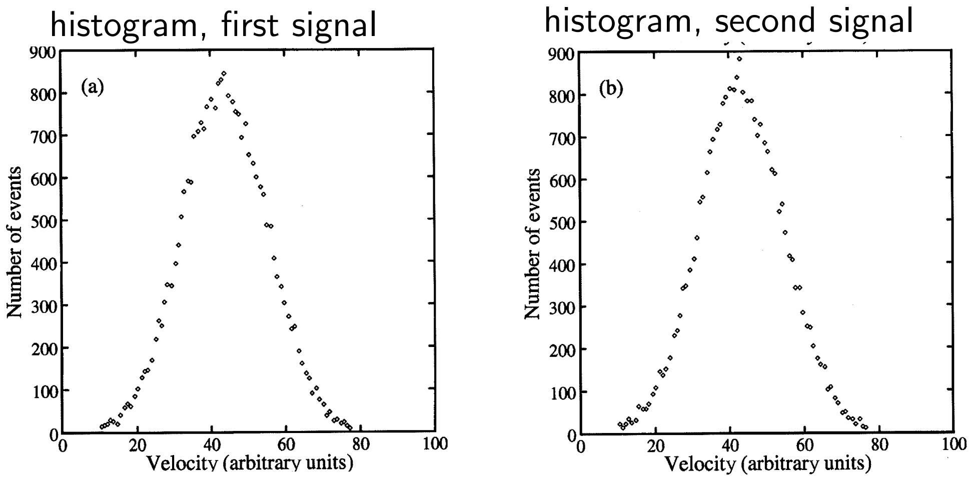


(Frisch “Turbulence”, 1995)

The random nature of turbulence (3)

Corresponding histograms

(Frisch "Turbulence", 1995)



$\Rightarrow$ statistical description makes sense (cf. lecture 3)

The sensitivity to initial conditions

- "butterfly effect" (Lorenz 1972)
- deterministic non-linear systems can exhibit chaotic behavior
- chaos/irregularity/disorder is not part of the equations
- examples for simple systems with chaotic behavior:
  - "logistics map": $x_{n+1} = a\,x_n(1 - x_n)$ chaotic for $a > 3.75$
  - double pendulum
  - point vortex systems:

$$\frac{\mathrm{d}x_j}{\mathrm{d}t} = -\frac{1}{2\pi}\sum_{i\neq j}^{n}\frac{\kappa_i(y_j - y_i)}{r_{ij}^2}\,, \qquad \frac{\mathrm{d}y_j}{\mathrm{d}t} = \frac{1}{2\pi}\sum_{i\neq j}^{n}\frac{\kappa_i(x_j - x_i)}{r_{ij}^2}$$

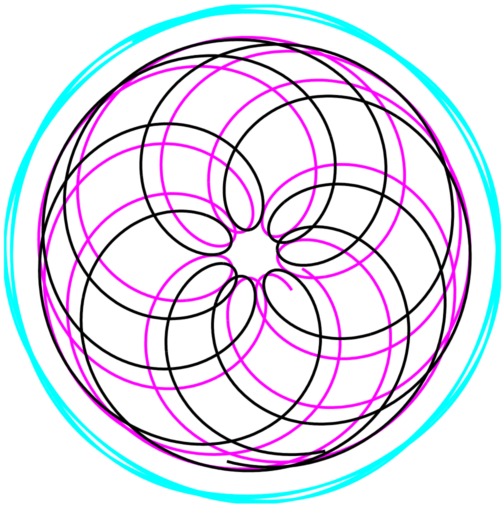
2D: chaotic for $n \geq 6$

$\Rightarrow$ initially close states diverge exponentially

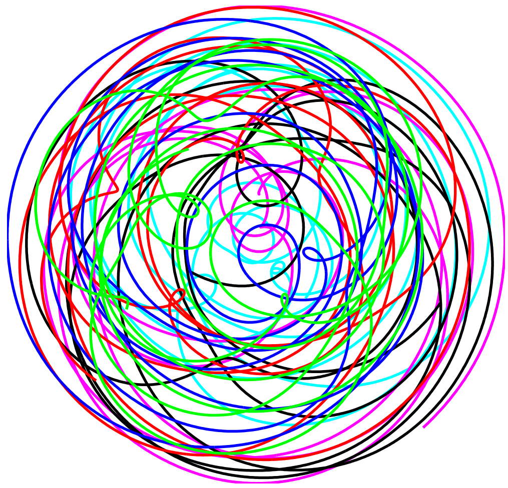
2D point vortex systems

Example trajectories from random initial positions

3 vortex system



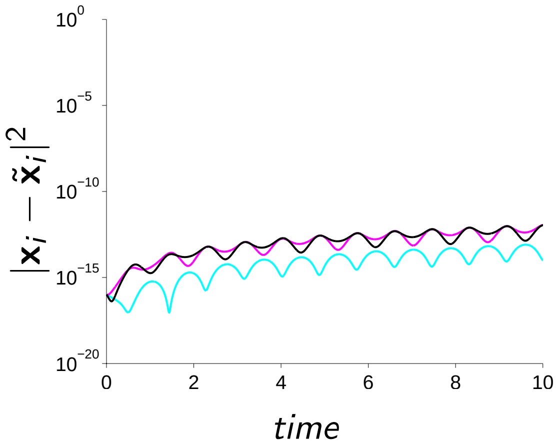
6 vortex system



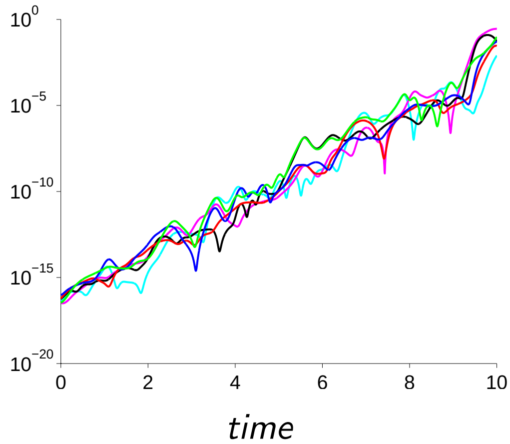
2D point vortex systems

Deviation of trajectories with time

3 vortex system



6 vortex system



$\longrightarrow$ "stable" solution

$\longrightarrow$ small perturbations *exp*-amplified

Conclusion from point vortex example

Point vortex dynamics

- deterministic non-linear equation can lead to chaotic behavior
- chaotic systems are extemely dependent on initial conditions

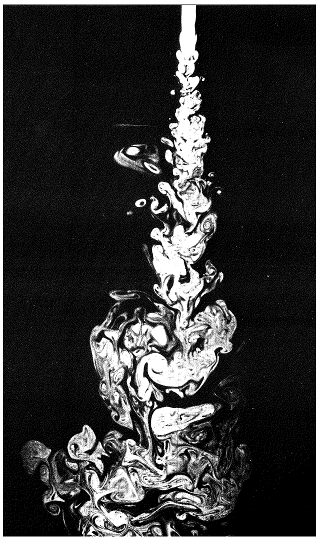
Similarly for the full Navier-Stokes equations

$\Rightarrow$ Statistical approach

Fundamental properties of turbulent flow

- irregularity:
  - chaotic motion in time and space (3D)

  but: averages are well-defined
- complex vortical motion
  - irrotational flow does not give rise to chaos
- wide range of scales
  - from size of the geometry to dissipative scales
- increased transport
  - momentum, mass, heat
- dissipative



Effects of turbulence can be desired or undesired

Effects of increased properties

	not desired	desired
momentum:	drag increase	delayed separation
mass:	controllability of processes	mixing in combustion, dispersion of pollutants
heat:	heat loss	efficient exchange

$\Rightarrow$ Turbulent flow control is an important field

- aeronautics, turbomachinery, ...

Also: stability, transition

Different approaches to computing turbulent flows (1)

Time-dependent, 3D solution of Navier-Stokes:

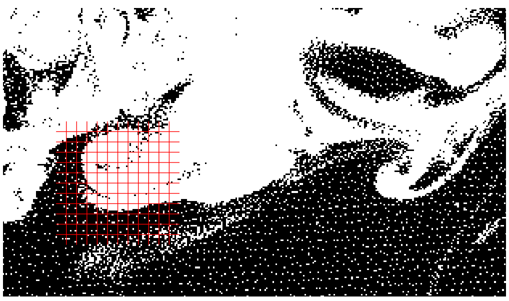
"Direct numerical simulation" (DNS)

- given the field $\mathbf{u}(t=0)$ and boundary conditions

$\longrightarrow$ integrate in time, then compute desired quantity

Problems:

- sensitivity to initial conditions $\longrightarrow$ statistics
- enormous computational requirements
- (cf. lecture 7)



Different approaches to computing turbulent flows (2)

Spatially filtered Navier-Stokes:

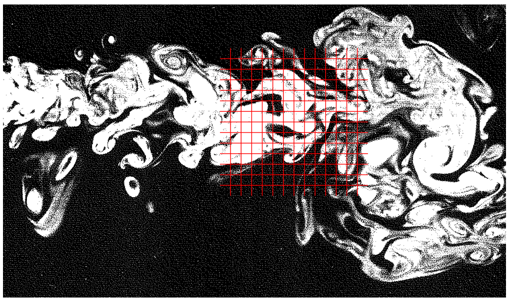
"Large eddy simulation" (LES)

- evolution of large scales is simulated
- small-scales are modelled

$\longrightarrow$ substantial savings w.r.t. DNS (coarser grid, larger step)

Problems:

- 3D, time-dependent
- costly for applications
- (cf. separate course on LES)



Different approaches to computing turbulent flows (3)

Reynolds-averaged Navier-Stokes (RANS)

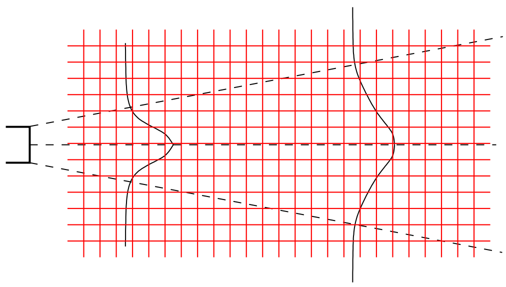
- equations are averaged *a priori*
- turbulence appears as additional unknowns

$\longrightarrow$ often stationary problem, additional symmetries

Problems:

- closure problem*

$\longrightarrow$ modelling
(cf. lectures 8…)



Different approaches to computing turbulent flows

- direct simulation (DNS)
- large-eddy simulation (LES)
- RANS

accuracy, simulation cost ↑

modeling complexity ↓

From fluid mechanics to turbulent flow computation

Stages of the analysis:

fluid mechanical process

$\Downarrow$ physics

mathematical model

$\Downarrow$ numerical maths

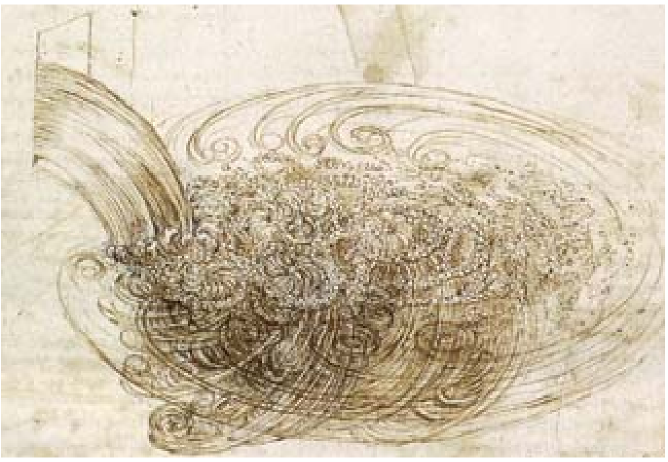
discrete equations

$\Downarrow$ coding

computational algorithm

Interdependence of the three stages

da Vinci on Turbulence



"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion." *(cited by Lumley, Phys. Fluids A, 1992)*

History of early turbulence research (rough list)

- 1510 da Vinci:
  observes eddying motion in water
- 1854 Hagen, Darcy:
  observe two different laws for pressure drop in pipes
- 1851-70 St. Venant, Boussinesq:
  introduce concept of eddy viscosity
- 1883-94 Reynolds:
  transition criterion in pipes, flow decomposition, stresses
- 1922 Richardson:
  formulation of cascade process
- 1941 Kolmogorov:
  quantitative theory for the cascade at high Re number
- and: Prandtl, Taylor, von Karman, ...

Summary

Main questions of the present lecture

- Why is fluid turbulence important?
  - ubiquity; greatly enhances transport (mass, momentum, heat)
- How do we define turbulence?
  - irregular, 3D, vortical, wide range of scales
- What are the principal difficulties for engineers?
  - unpredictable in detail, direct simulation too costly
  - LES, RANS approach more feasible

$\Rightarrow$ need for closure models

Outlook on next lecture: Equations of fluid motion

How can the fluid motion be described mathematically?

What are the transformation properties of the conservation laws?

Film "Turbulence" by R.W. Stewart

Further reading

- M. van Dyke, *An Album of Fluid Motion*, 1982
  $\longrightarrow$ visualization of interesting flow physics
- U. Frisch, *Turbulence*, 1995
  $\longrightarrow$ chapter 1 for an intruduction to turbulence and symmetries