Turbulenzmodelle in der Strömungsmechanik

Turbulent flows and their modelling

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LECTURE 11

Boundary conditions and wall treatment

Questions to be answered in the present lecture

How can RANS models be applied to wall-bounded flows?

- the wall-function approach
- specific model modifications for the wall region
- elliptic relaxation models for the pressure-strain correlation

Physical effects in turbulent wall-bounded flow

Main features of the near-wall region:

1. (locally) low Reynolds number $Re_L \equiv k^2/(\varepsilon\nu)$
2. high shear-rate $\mathcal{S}k/\varepsilon$
3. tendency towards two-component turbulence
4. wall-blocking effect on the pressure field

Consequences for modeling:

$\rightsquigarrow$ basic RANS models are not applicable

$\Rightarrow$ most models need special ajustment

Basic approaches for modeling near-wall flow

Wall-function approach

- *bridging* the viscous wall region with analytical functions

Specific models for the wall region

- various *modifications* applied to the original formulations

Elliptic relaxation models

- *non-local* model for pressure-strain correlation

Damping turbulent viscosity in k-ε models

Accounting for erroneous near-wall behavior

- overprediction with original expression: $\nu_T = C_\mu \frac{k^2}{\varepsilon}$
- *damped* turbulent viscosity formulas:

$$\nu_T = f_\mu C_\mu \frac{k^2}{\varepsilon} \qquad (0 \leq f_\mu(y) \leq 1)$$

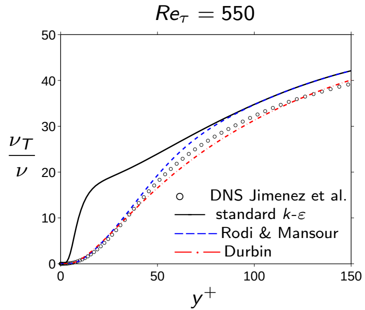
- e.g. Rodi & Mansour (1993) propose *data fit*:

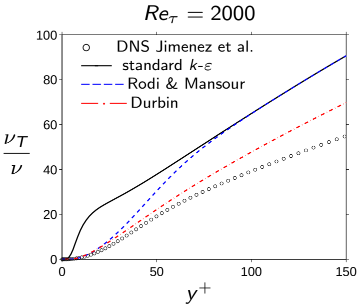
  $f_\mu = 1 - \exp\left(-0.0002\, y^+ - 0.00065\, y^{+2}\right)$
- Durbin (1991) considers *wall-normal stress* responsible:

  (v' reduced near wall w.r.t. u', w' – cf. lecture 6)

  $\rightarrow \nu_T = C'_\mu \langle v'v' \rangle \frac{k}{\varepsilon}$ with: $C'_\mu = 0.22$

Turbulent viscosity in plane channel flow





- modifications based on damping function f_μ lack generality

$\rightsquigarrow$ they fail in BL with pressure gradient; numerically stiff

- Durbin's $\langle v'v' \rangle$-expression for ν_T more physical

$\rightsquigarrow$ Reynolds dependence of ν_T: C'_μ should be fct. of Reynolds

Near-wall modifications to the dissipation rate

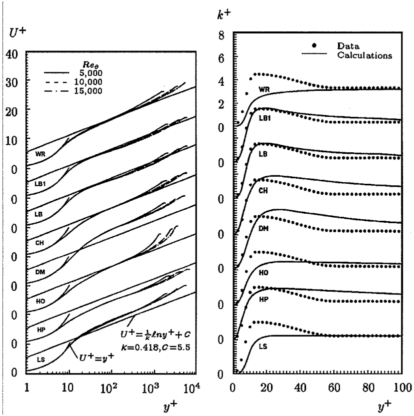
The dissipation rate equation

- the standard model uses completely artificial equation
- not directly applicable to near-wall region
- instead:
$$\frac{\bar{\mathrm{D}}\hat{\varepsilon}}{\bar{\mathrm{D}}t} = \left(\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\hat{\varepsilon}_{,j}\right)_{,j} + f_1 C_{\varepsilon_1}\mathcal{P}\frac{\hat{\varepsilon}}{k} - f_2 C_{\varepsilon_2}\frac{\hat{\varepsilon}^2}{k} + E$$
- with: $\hat{\varepsilon} = \tilde{\varepsilon} - \nu\partial_{yy}k)_{y=0}$ $\rightarrow$ such that $\hat{\varepsilon}(y=0)=0$
- modifications through functions f_1, f_2, E
- various proposals exist (e.g. Jones & Launder, 1972; Launder & Sharma, 1974; Lam & Bremhorst 1981)

Performance of various modified k-ε models

Flat-plate BL flow

- ⇝ large discrepancies
- ⇝ also: poor performance in adverse pressure gradient



(Patel et al. 1985)

⇒ most near-wall k-ε models are *not* reliable!

Alternative two equation models for wall-bounded flow

Wilcox' 1993 k-ω model (cf. lecture 9)

$$\frac{\bar{\mathrm{D}}k}{\bar{\mathrm{D}}t} = \left(\left(\nu + \frac{\nu_T}{\sigma_k}\right)k_{,j}\right)_{,j} + \mathcal{P} - C_\mu k\omega$$

$$\frac{\bar{\mathrm{D}}\omega}{\bar{\mathrm{D}}t} = \left(\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\omega_{,j}\right)_{,j} + \mathcal{P}\frac{C_{\omega_1}\omega}{k} - C_{\omega_2}\omega^2$$

with: $\nu_T = k/\omega$, and model parameters: $\sigma_\omega, C_{\omega_1}, C_{\omega_2}$

- no wall-damping necessary

$\rightsquigarrow$ but: high sensitivity to freestream boundary conditions

$\rightsquigarrow$ use of the original model *problematic* in general flows!

Menter's 1994 SST model

1. Blending the ω and ε equations

- ω equation derived from $\tilde{\varepsilon}$: cross-diffusion term $S_\omega = k_{,j}\omega_{,j}/\omega$
- SST model: use $S_\omega\,(1 - F_1)$ instead $\qquad (0 \leq F_1(y) \leq 1)$

$\Rightarrow$ k-ω model near wall ($F_1 \to 1$), k-ε in outer region ($F_1 \to 0$)

2. Limiting the value of turbulent viscosity in BL

- k-ω overpredicts shear stress in adverse pressure gradient
- limiter in SST model: $\qquad \nu_T = \min\left(\frac{k}{\omega}, \frac{\sqrt{C_\mu}\,k}{2|\bar{\mathbf{\Omega}}|}\frac{1}{F_2}\right)$

$\Rightarrow$ assures that $|\langle u'v'\rangle|/k$ smaller than $\sqrt{C_\mu} = 0.3$

- $F_2 \to 0$ outside BL $\qquad$ ($\mathbf{\Omega}$ is mean rate-of-rotation)

Performance of the SST model

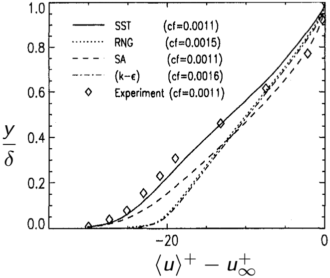
Flat-plate BL with adverse pressure gradient

- adverse pressure gradient:

$$\beta \equiv \frac{\delta_*}{\tau_w}\frac{\mathrm{d}p}{\mathrm{d}x} = 8.7$$

- SST model yields improved predictions
- works also reasonably well in separated flows

mean velocity



(from Menter 1996)

The modified k-ω model of Wilcox (2006)

New k-ω model

- contains a limiter function in ν_T

  $\longrightarrow$ avoids overpredicting shear stress in APG

- uses a localized cross-diffusion term

  $\longrightarrow$ reduces free-stream sensitivity of the original model

$\Rightarrow$ formulation is similar to SST model

$\rightsquigarrow$ no direct comparison of k-ω versus SST models available in literature

The wall function approach

Two-fold problem of computations in near-wall region

- modeling complexities
- high computational effort due to steep gradients

Possible trick

- if flow is approximately parallel to surface:
  $\longrightarrow$ skip near-wall region
  $\longrightarrow$ compute only from log-region outwards

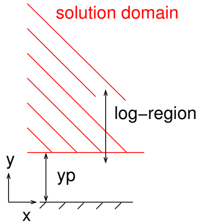
$\Rightarrow$ apply boundary conditions in log-region

Deriving wall-functions for the k-ε model

Conditions for equilibrium boundary layer

- suppose $y = y_p$ located in the log-region
- log-law for mean streamwise velocity:
  $\langle u \rangle = u_\tau \left(\frac{1}{\kappa} \log(\frac{y_p u_\tau}{\nu}) + B \right)$
- approximate log-layer relations:
  $\mathcal{P} = \tilde{\varepsilon}, \qquad -\langle u'v' \rangle = u_\tau^2$

$\Rightarrow \; \tilde{\varepsilon} = \frac{u_\tau^3}{\kappa y_p}, \qquad k = \frac{u_\tau^2}{\sqrt{C_\mu}}$



Note for Reynolds stress models

- the stresses are approximately constant:

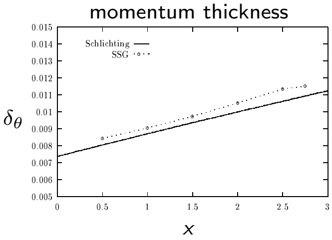
  $\partial_y \langle u_i' u_j' \rangle = 0$

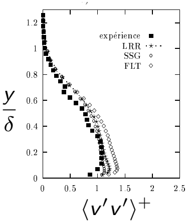
Wall functions – practical implementation (example)

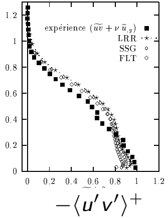
Given current values: $\langle u \rangle_p = \langle u \rangle(y_p)$, $k_p = k(y_p)$

1. compute friction velocity: $u_\tau^* = C_\mu^{1/4} k_p^{1/2}$
2. compute nominal velocity: $\langle u \rangle^* = u_\tau^* \left(\frac{1}{\kappa} \log(\frac{y_p u_\tau^*}{\nu}) + B \right)$
3. compute shear stress: $-\langle u' v' \rangle^* = u_\tau^{*2} \frac{\langle u \rangle_p}{\langle u \rangle^*}$
4. apply shear stress as boundary flux on streamwise momentum
   $\longrightarrow$ this provides robust condition
5. apply zero-gradient condition on k
6. impose: $\tilde{\varepsilon}^* = \frac{u_\tau^{*3}}{\kappa y_p}$

Results obtained with RSM and wall functions







Zero-pressure-gradient boundary layer at $Re_\theta = 7600$

- wall-function computations yield predictions in agreement with measurements (here: exp. of Klebanoff 1954)

Extending the wall function approach

Various additional effects can be incorporated:

- roughness: variation of intercept constant B in log-law
- if boundary point is located *below* the logarithmic layer:
  $\rightarrow$ approximate expression for viscous sublayer (Durbin 2001)
- incorporating effect of pressure gradients (Shih et al. 1999)
- surface heat transfer effects (Nichols & Nelson 2004)
- ...

Problems with the wall function approach

Complex wall-bounded flows: no log-layer exists!

- separated flows
- impinging jets
- transitional boundary layers

$\rightsquigarrow$ wall-functions physically unrealistic!

Reynolds stress models: Anisotropy of dissipation

Observations (cf. lecture 10)

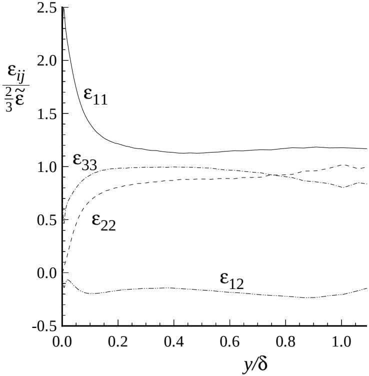
- high Reynolds number *and* far from solid boundaries:

  $\longrightarrow$ approximate isotropy

  $\varepsilon_{ij} \approx \frac{2}{3}\tilde{\varepsilon}\,\delta_{ij}$

- near the wall:

  $\longrightarrow$ significant anisotropy



(DNS Spalart 1988, $Re_\theta = 1410$)

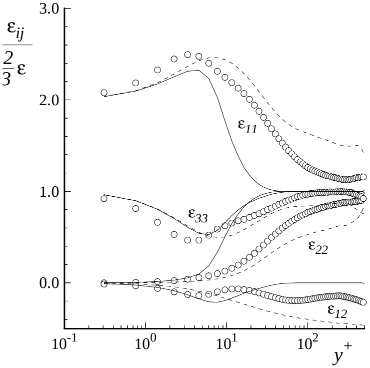
Models for near-wall anisotropy of dissipation

- asymptotically for $y \longrightarrow 0$:

$$\begin{aligned} \varepsilon_{ij}/\varepsilon &= \langle u_i' u_j' \rangle / k \quad i \neq 2, j \neq 2 \\ \varepsilon_{i2}/\varepsilon &= 2\langle u_i' u_j' \rangle / k \quad i \neq 2 \\ \varepsilon_{22}/\varepsilon &= 4\langle u_i' u_j' \rangle / k \end{aligned}$$

- Rotta (1951) model:
  $\varepsilon_{ij}/\varepsilon = \langle u_i' u_j' \rangle / k$
- Lai & So (1990) model:
  blending with isotropic expression for outer flow
  $\varepsilon_{ij} = \varepsilon_{ij}^* f_s + (1 - f_s)\frac{2}{3}\delta_{ij}\varepsilon$

$\rightsquigarrow$ blending too rapid



$\Rightarrow$ more elaborate models exist

Wall-effects on the pressure field

Poisson equation for fluctuating pressure

$\nabla^2 p' = S_p$ with: $S_p \equiv -2\rho\langle u_i\rangle_{,j}\, u'_{j,i} - \rho\left(u'_i u'_j - \langle u'_i u'_j\rangle\right)_{,ij}$

- decomposition: $p' = p^{(h)} + p^{(p)}$
- *homogeneous* pressure $p^{(h)}$: $\nabla^2 p^{(h)} = 0$
- *particular* pressure $p^{(p)}$: $\nabla^2 p^{(p)} = S_p$
- $p^{(h)} = 0$ in *unbounded* flow

Effects in wall-bounded flow:

1. $p^{(h)} \neq 0$
2. $p^{(p)}$ modified due to wall reflection

Wall-effects on the pressure field (2)

Effect of non-zero homogeneous pressure $p^{(h)}$

- boundary condition at a wall: $\partial_y p'/\rho = \nu \partial_{yy} v'$
- DNS shows: small effect, negligible for $y^+ \geq 15$

$\Rightarrow$ $p^{(h)}$ contribution to pressure-strain correlation neglected in RANS models

Wall-effects on the pressure field (3)

Modification of particular pressure $p^{(p)}$

- appropriate boundary condition at wall: $\partial_y p^{(p)} = 0$

$$\Rightarrow \quad p^{(p)}(\mathbf{x}) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_p(\mathbf{x}')\frac{1}{|\mathbf{x}-\mathbf{x}'|}\mathrm{d}\mathbf{x}'$$

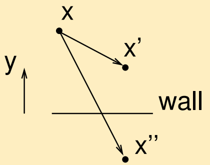
where source is *mirrored* at wall: $S_p(-y) = S_p(y)$

- the solution can be rewritten as:

$$p^{(p)}(\mathbf{x}) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} S_p(\mathbf{x}')\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}+\frac{1}{|\mathbf{x}-\mathbf{x}''|}\right)\mathrm{d}\mathbf{x}'$$

- "wall-echo" effect

$\Rightarrow$ pressure-strain has additional contribution



Wall-echo pressure-strain modeling

Consequences for RSM modeling

- the additional wall-echo term decays as y_w^{-1}
- some pressure-strain models incorporate *explicit* terms for wall-echo effects, e.g. Gibson & Launder (1978):

$$\mathcal{R}_{ij}^{(s,w)} = 0.2\frac{\varepsilon}{k}\frac{L}{y_w}\left(\langle u_l u_m\rangle n_l n_m \delta_{ij} - \frac{3}{2}\langle u_i u_l\rangle n_j n_l - \frac{3}{2}\langle u_j u_l\rangle n_l n_i\right)$$

$\rightsquigarrow$ *problem:* wall-echo models lack universality

e.g. deteriorated performance in impinging flows

Elliptic relaxation models

- Durbin (1993): problem is *local* expression for pressure-strain
- proposes elliptic partial differential equation:

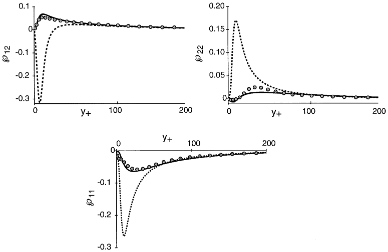
  $$f_{ij} - L_D^2 \nabla^2 f_{ij} = \mathcal{R}_{ij}/k, \qquad \mathcal{R}_{ij}^{(e)} = f_{ij}\, k$$

  with: $\mathcal{R}_{ij}$ usual redistribution model; L_D a length scale
- *homogeneous* flow: $\nabla^2 f_{ij} = 0 \quad \longrightarrow \quad \mathcal{R}_{ij}^{(e)} = \mathcal{R}_{ij}$
- *otherwise:* operator $(I - L_D^2 \nabla^2)$ mimics non-localness of pressure-Poisson equation

$\Rightarrow$ takes into account boundary conditions on f_{ij}

$\rightsquigarrow$ note: 6 additional PDEs added to the system

Performance of elliptic relaxation models



(from Durbin & Petterson Reif, 2001)

redistribution terms:

$\Pi_{ij} - \varepsilon_{ij} + \frac{2}{3}\tilde{\varepsilon}\delta_{ij}$

○	DNS Mansour et al. (1988)
– – – –	SSG model
——	SSG/elliptic relaxation

Near-wall region of plane channel flow:

- elliptic relaxation models with RSM can significantly improve predictions of redistribution terms

Summary

How can RANS models be applied to wall-bounded flows?

1. the wall-function approach
   - allows to bridge near wall region with analytic expressions
   - $\rightsquigarrow$ problems in non-parallel flows (separation)
2. modified two-equation models for the wall region
   - $\rightsquigarrow$ most k-ε-based models lack generality
   - k-ω and SST models yield better performance
3. modifications to Reynolds-stress models
   - $\rightsquigarrow$ distance-function-based approach lacks generalit (dissipation tensor, pressure wall-echo)
   - elliptic relaxation promising for complex wall-bounded flows

Outlook: Algebraic stress models

How can the linear eddy viscosity assumption be avoided *without* the need for solving transport equations?

- algebraic stress models
- nonlinear eddy viscosity models

Further reading

- S. Pope, *Turbulent flows*, 2000
  $\longrightarrow$ chapter 11
- P.A. Durbin and B.A. Pettersson Reif,
  Statistical theory and modeling for turbulent flows, 2003
  $\longrightarrow$ chapter 7
- D.C. Wilcox, *Turbulence modeling for CFD*, 2006
  $\longrightarrow$ chapter 4 and 6